

# FBP 2021

15th International Conference on Free Boundary Problems: Theory and Applications

Berlin, Germany, September 13–17, 2021

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## Welcome Address

Welcome to the 15th International Conference on Free Boundary Problems: Theory and Applications!

This edition of the conference is special in several respects. First and foremost, FBP 2021 has attracted an excellent group of plenary speakers, it offers a list of minisymposia on very timely topics, and it has received very interesting contributed presentations. This is clearly a sign that the community is very active and thriving. On the other hand, FBP 2021 stands out of the conference series as it will be held fully online for the first time ever, owing to the consequences of the COVID pandemic. Needless to say that the local organizing team and the Berlin mathematical community would have preferred an in-person meeting at Humboldt-Universität zu Berlin, as originally planned for 2020. Indeed, gathering a group of esteemed colleagues at a place for intense scientific exchange is always rewarding in many respects. In order to cope as best as possible with the challenges of holding an international meeting online, the organizing team has tried to adjust the schedule to the various time zones and employ electronic tools which admit ad hoc discussions and personal encounters. With these measures in place and given our experience as a community with virtual meetings, the organizing team as well as the program committee are confident that FBP 2021 will add another highlight to this traditional conference series.

Overall, we are looking forward to about 150 invited and contributed talks and 12 invited plenary lectures. The electronically available conference book has been designed to provide quick information and access to virtual presentation rooms. Moreover, we hope that you will use it to select not only minisymposia sessions that you find interesting to attend but also that you will peruse talks by our excellent contributed speakers. Our warm gratitude goes to the minisymposia organizers and speakers as well as the contributed speakers without whom this conference could not exist.

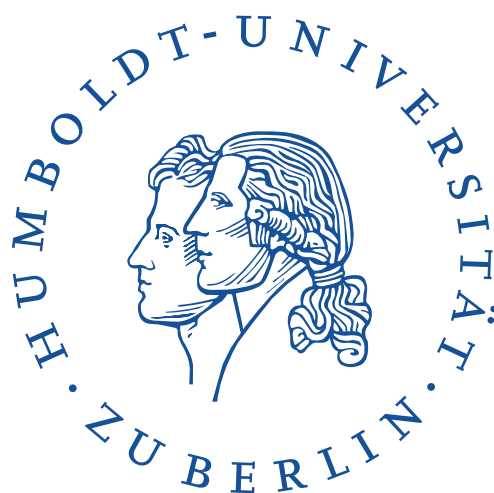
We wish you all a pleasant and memorable FBP 2021, and a lot of exciting mathematics!

FBP 2021 is Supported by

**DFG** Deutsche  
Forschungsgemeinschaft



**Weierstrass Institute for  
Applied Analysis and Stochastics**



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## Plenary Speakers and Minisymposia Organizers

## Plenary speakers

SÖREN BARTELS  
U Freiburg

QIANG DU  
Columbia U

XAVIER ROS-OTON  
U Zurich

ANDREA BERTOZZI  
UCLA

ALESSIO FIGALLI  
ETH Zurich

ROBERT SAYE  
Lawrence Berkeley Lab

LIA BRONSARD  
McMaster U

IRENE FONSECA  
Carnegie Mellon U

YOSHIHIRO TONEGAWA  
Tokyo Institute of Technology

ANTONIN CHAMBOLLE  
U Paris-Dauphine PSL

BARBARA NIETHAMMER  
U Bonn

ENRIQUE ZUAZUA  
U Erlangen

## Minisymposia

**Nonlocal free boundary problems**

J.-F. Rodrigues, E. Valdinoci

**Free boundary problems related to shapes and geometries**

G. Buttazzo, E. Oudet

**Regularity of free boundaries**

H. Shahgholian, M. Smit Vega Garcia

**Phase field models**

E. Rocca, V. Styles

**Optimization and control of FBPs**

M. Hintermüller, M. Hinze

**Numerical methods for geometric PDEs**

R. Nochetto, R. Nürnberg

**Surface PDEs**

C. Venkataraman, T. Ranner

**Free boundary models in active matter**

L. Berlyand

**Numerical methods for surface-bulk problems**

E. Bänsch, A. Reusken

**Interfaces in fluids**

H. Abels, H. Garcke, M. Wilke

**FBP in the life sciences**

L. Berlyand, A. Friedman

**Free boundary problems for nonlinear hyperbolic/mixed-type PDEs**

G.-Q. Chen

**Stochastic free boundary problems**

A. Djurdjevac, B. Gess, G. Grün

**Free boundary problems in cell biology**

C. Gräser, M. Röger

**Asymptotic approaches to interface dynamics**

J. José López Velázquez, J. King

**Geometric problems**

Y. Giga, M. Novaga

**UQ in free boundary problems**

H. Harbrecht, M. D. Multerer

**Geometric evolution of interfaces and transition layers**

(contributed minisymposium)

P. Rybka, G. Wheeler

## Biographies of Plenary Speakers

**Sören Bartels** (University of Freiburg) is an applied mathematician whose interests lie in the development and analysis of approximation schemes for PDEs with applications in material science and geometry. He is also the author of several books related to numerical methods for PDEs.

**Andrea Bertozzi** (University of California, Los Angeles) is an applied mathematician with expertise in nonlinear partial differential equations and fluid dynamics. She also works in the areas of geometric methods for image processing, crime modeling and analysis, and swarming/cooperative dynamics. Bertozzi completed all her degrees in mathematics at Princeton. Since 2005 she has served as Director of Applied Mathematics, overseeing the graduate and undergraduate research training programs at UCLA. In May 2018 Bertozzi was elected to the US National Academy of Sciences. In July 2019 she was awarded SIAM's Kleinman Prize, which recognizes contributions that bridge the gap between high-level mathematics and engineering problems.

**Lia Bronsard** (McMaster University) is originally from Québec and did her undergraduate studies at the Université de Montréal, graduating in 1983. She earned her PhD in 1988 from New York University under the supervision of Robert V. Kohn. After short-term positions at Brown University, the Institute for Advanced Study, and Carnegie Mellon University, she moved to McMaster University in 1992. She was president of the Canadian Mathematical Society for 2014–2016. Bronsard was a plenary speaker at the SIAM Annual meeting in Boston in 2016, at the Mathematical Congress of the America in Montreal in 2017, and at the CMS Summer meeting in Charlottetown in 2018. She is on the editorial board of *Nonlinear Analysis*, *Mathematics in Science and Industry*, and the *Canadian Applied Math Quarterly*. In her research, she has used geometric flows to model the interface dynamics of reaction–diffusion systems. Other topics in her research include pattern formation, grain boundaries, and vortices in superfluids. Bronsard was the 2010 winner of the Krieger–Nelson Prize. In 2018 the Canadian Mathematical Society listed her in their inaugural class of fellows.

**Antonin Chambolle** (CNRS and Université de Paris-Dauphine PSL) is a CNRS research director at CEREMADE, Université de Paris-Dauphine PSL (France). His research focuses on free boundaries or free discontinuity problems, variational methods in image processing, variational brittle fracture theory in elasticity and continuous (convex) optimization.

**Qiang Du** (Columbia University) is the Fu Foundation Professor of Applied Mathematics at the Department of Applied Physics and Applied Mathematics (APAM), Fu Foundation School of Engineering and Applied Science (SEAS), Columbia University. He is also affiliated with the Data Science Institute (DSI). He chaired the Applied Mathematics Program Committee from 2014–2020 and served in DSI as a co-Chair of the Center for Foundations of Data Science from 2017–2019. Currently he also serves as a co-chair of the Center for Computing Systems for Data-Driven Science.

**Alessio Figalli** (ETH Zürich) works in the theory of optimal transportation, elliptic PDEs, free boundaries, and functional inequalities. He has also given several contributions to the theory of transport equations with rough vector-fields, Hamilton-Jacobi equations, and random matrices. He received the Fields Medal in 2018.

**Irene Fonseca** (Carnegie Mellon University) received her Ph.D. degree in mathematics in 1985 from the University of Minnesota, has been a faculty member in the Mellon College of Science since 1987 and is the director of the university's Center for Nonlinear Analysis. She has supervised 16 Ph.D. students and mentored 41 postdoctoral fellows. One of the world's leading researchers in the field of applied mathematics, Fonseca's research lies at the interface of applied analysis with materials and imaging sciences. In 2012, she became the second woman to be elected president of the Society for Industrial and Applied Mathematics (SIAM). She is a Fellow of the American Mathematical Society (AMS) and of SIAM. She is a Grand Officer of the Military Order of Saint James of the Sword (Grande Oficial da Ordem Militar de Sant'Iago da Espada, Portuguese Decoration). She received the Mellon College of Science chair in 2003, in 2014 she was named University Professor, the highest distinction that can be bestowed on a professor at CMU, and in 2018 she was endowed the Kavčić–Moura chair in mathematics. Fonseca serves in 18 editorial boards, including *Advances in Calculus of Variations*, *Archive for Rational Mechanics and Analysis*, *ESAIM:COCV* (SMAI), *Journal of Nonlinear Science*, *M3AS*, and *SIAM Journal on Mathematical Analysis*. She is a member of several advisory and scientific boards of research centers and institutes, including the IMA, she participates in international prize committees, and is in review and evaluation panels of multiple universities in the US and abroad.

**Barbara Niethammer** (University of Bonn) is a professor at the Hausdorff Center for Mathematics at the University of Bonn. Her research interests are in applied mathematics and include the analysis of problems with multiple scales, dynamics in high-dimensional dynamical systems and universal scaling behavior in models of mass aggregation and coarsening. Niethammer won the Richard von Mises Prize of the Gesellschaft für Angewandte Mathematik und Mechanik in 2003 for her work on Ostwald ripening, and the Whitehead Prize of the London Mathematical Society in 2011 "for her deep and rigorous contributions to material science, especially on the Lifshitz–Slyozov–Wagner and Becker–Doering equations".

**Xavier Ros-Oton** (University of Barcelona) has made outstanding contributions to the analysis of various aspects of elliptic and parabolic PDEs, including free boundary problems. He has won numerous awards for his achievements by various societies/institutes in Spain, including being chosen as the winner of the prestigious Premio Investigación Científica 2019 award (granted by the king of Spain) for young Spanish mathematicians. He is a PI of a ERC Starting Grant where he was the youngest awardee amongst all starting grants in 2018 across all the sciences. His collaborators include exemplary figures in the free boundary problem community such as Luis Caffarelli.

**Robert Saye** (Lawrence Berkeley National Laboratory) is an applied mathematician in the Mathematics Group of the Computational Research Division at the Lawrence Berkeley National Laboratory and is also affiliated with the Department of Mathematics of the University of California at Berkeley. His research interests are in mathematical modelling and scientific computation, particularly numerical methods for interfacial fluid dynamics, high-order accurate algorithms for implicitly defined geometry, and multi-scale and multi-physics simulation.


**Yoshihiro Tonegawa** (Tokyo Institute of Technology) is an expert working in elliptic and parabolic PDEs as well as geometric measure theory and curvature flows. The quality of his work was recognised in 2010 when he won the Fukuhara Prize of the Mathematical Society of Japan and once again in 2013 when he won the society's Analysis Prize. He is also the author of a book on Brakke's mean curvature flow.

**Enrique Zuazua** (University of Erlangen–Nuremberg) is Chair Professor in Applied Analysis — Alexander von Humboldt Professorship at the University of Erlangen–Nuremberg (FAU). He is also the Principal Investigator of the European Research Council (ERC) Advanced Grant “DyCon - Dynamic Control”, at the Chair of Computational Mathematics of Deusto Foundation in Bilbao and Universidad Autónoma de Madrid (UAM). His domains of expertise in applied mathematics include partial differential equations, control theory, numerical analysis, and data sciences. These subjects interrelate with the final to model, analyse, compute, simulate, and finally contribute to the control and design of the most diverse natural phenomena and all fields of R & D.

### A communication from Henrik Shahgholian

The free web-based platform called **SciLag**, accessible at <https://scilag.net>, provides tools for mathematicians to collaboratively build a dynamic and tractable database of open problems in mathematics, allowing researchers to announce, discover, and discuss open problems stemming from their research. A few examples of our current problems are:

- PDE: <https://www.scilag.net/problem/P-180630.2>
- Group Theory: <https://www.scilag.net/problem/P-180619.3>
- Probability: <https://www.scilag.net/problem/P-181120.2>
- Diff. Geometry: <https://www.scilag.net/problem/P-180904.1>
- Combinatorics/PDE: <https://www.scilag.net/problem/P-181023.1>
- Combinatorics: <https://www.scilag.net/problem/P-181012.1>

The following pdf file  illustrates a Scilag-page for registered members.

## Daily Overview

Monday 13.09.2021

08:45–09:00: Opening

Lecture hall

09:00–09:45: Plenary

**Discretizations of the total variation for singular functions**

Antonin Chambolle

Lecture hall  
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09:50–11:50: Parallel Sessions

**Interfaces in fluids (Part I)**, H. Abels, H. Garcke, M. Wilke

Christina Lienstromberg

Elisabetta Rocca

Piotr B. Mucha

Room 1

Marius Tucsnak

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**Optimization and control of FBPs (Part I)**, M. Hintermüller, M. Hinze

Christian Kahle

Tobias Keil

Kei Fong Lam

Room 2

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**Numerical methods for surface-bulk problems (Part I)**, A. Reusken, E. Bänsch

Markus Gahn

André Massing [canceled]

Dirk Peschka

Room 3

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12:45–14:45: Parallel Sessions

**Free boundary problems related to shapes and geometries**, E. Oudet, G. Buttazzo

Giuseppe Buttazzo

Edouard Oudet

Bozhidar Velichkov

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**Phase field models (Part I)**, E. Rocca, V. Styles

Klaus Deckelnick

Pierluigi Colli

Michael Hinze

Room 2

Maurizio Grasselli

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15:00–15:45: Plenary

**Phase transitions in heterogeneous media: equilibria and geometric flows**

Irene Fonseca

Lecture hall  
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16:15–17:00: Plenary

**Higher-Order Numerics for Interfacial Fluid Dynamics**

Robert Saye

Lecture hall  
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Wednesday 15.09.2021

09:00–09:45: Plenary

<b>Simulation of Free Boundary Problems in the Nonlinear Bending of Elastic Rods and Plates</b>	Lecture hall
Sören Bartels	page 14

09:50–11:50: Parallel Sessions

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Philip Brandner	page 29
Argyrios Petras	Marino Arroyo
<b>Nonlocal free boundary problems (Part I)</b> , E. Valdinoci, J.-F. Rodrigues	Room 2
Alexander Nazarov	page 29
Juan Luis Vázquez	Xavier Fernández-Real
Aram Karakhanyan	Hubertus Grillmeier
<b>Stochastic free boundary problems (Part I)</b> , A. Djurdjevac, B. Gess, G. Grün	Room 3
Lubomir Banas	page 30
Sebastian Hensel	Nicolas Dirr
<b>Geometric problems (Part II)</b> , M. Novaga, Y. Giga	Room 4
Paola Pozzi	page 31
Mathias Wilke	Ulisse Stefanelli
Lucia Scardia	

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Nicola Soave	
<b>Numerical methods for surface-bulk problems (Part II)</b> , A. Reusken, E. Bänsch	Room 2
Stefan Metzger	page 32
Alfred Schmidt	Robert Nürnberg
Thomas Ranner	
<b>Optimization and control of FBPs (Part II)</b> , M. Hintermüller, M. Hinze	Room 3
René Pinnau [canceled]	page 33
Björn Baran	Dmytro Strelnikov
Patrik Knopf	
<b>Interfaces in fluids (Part III)</b> , H. Abels, H. Garcke, M. Wilke	Room 4
Andrea Giorgini	page 34
Ian Tice	Maurizio Grasselli

15:00–15:45: Plenary

<b>Nonlocal variational geometric problems with free boundary</b>	Lecture hall
Qiang Du	page 15

16:15–18:15: Parallel Sessions

<b>Numerical methods for geometric PDEs (Part II)</b> , R. Nochetto, R. Nürnberg	Room 1
Antonin Chambolle	page 35
Martin Rumpf	Klaus Deckelnick
Hanne Hardering	
<b>Free boundary models in active matter</b> , L. Berlyand	Room 2
Leonid Berlyand	page 35
Carles Blanch-Mercader	Ricard Alert
Jaume Casademunt	
<b>Regularity of free boundaries (Part II)</b> , H. Shahgholian, M. Smit Vega Garcia	Room 3
Mark Allen	page 36
Murat Akman	Dennis Kriventsov
Edgard Pimentel	

Thursday 16.09.2021

09:00–09:45: Plenary

<b>An obstacle problem for cell polarization</b> Barbara Niethammer	Lecture hall page 15
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09:50–11:50: Parallel Sessions

<b>Asymptotic approaches to interface dynamics (Part I)</b> , J. José López Velázquez, J. King John Ward Scott McCue	Amrita Ghosh Diego Alonso Oran	Room 1 page 37
<b>Stochastic free boundary problems (Part II)</b> , A. Djurdjevac, B. Gess, G. Grün Peter Nejjar	Benjamin Seeger Ana Djurdjevac	Room 2 page 38

12:45–14:45: Parallel Sessions

<b>Free boundary problems for nonlinear hyperbolic/mixed-type PDEs (Part II)</b> , G.-Q. Chen Mikhail Feldman Steve Shkoller	Paolo Secchi Dehua Wang	Room 1 page 39
<b>Free boundary problems in cell biology (Part I)</b> , C. Gräser, M. Röger Helmut Abels Anna Logioti	Martin Burger Luca Lussardi	Room 2 page 39
<b>Nonlocal free boundary problems (Part II)</b> , E. Valdinoci, J.-F. Rodrigues Lisa Santos Harbir Antil	Eduardo Teixeira Catharine W.K. Lo	Room 3 page 40

15:00–15:45: Plenary

<b>Generic regularity in obstacle problems</b> Alessio Figalli	Lecture hall page 16
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16:15–18:15: Parallel Sessions

<b>FBP in the life sciences</b> , A. Friedman, L. Berlyand Avner Friedman Tim Laux	Bei Hu Adrian Lam	Room 2 page 41
<b>Geometric evolution of interfaces and transition layers (Part I)</b> , G. Wheeler, P. Rybka Marco Morandotti Mikołaj Sierżęga	Danielle Hilhorst Katarzyna Ryszewska	Room 1 page 42
<b>Contributed session (Part IV)</b> Rolf Krause Lucas Daniel Wittwer	Irina Denisova Sebastian Aland	Room 3 page 42
<b>Contributed session (Part V)</b> Guozhi Dong	Vladimir Vasilyev Seongmin Jeon	Room 4 page 43

18:20–19:05: Plenary

<b>Energy Minimizing Surface Tension Configurations for Microparticles</b> Andrea Bertozzi	Lecture hall page 16
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Friday 17.09.2021

09:00–09:45: Plenary

**Inverse design for conservation laws and Hamilton-Jacobi equations**

Enrique Zuazua

Lecture hall

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09:50–11:50: Parallel Sessions

**Geometric evolution of interfaces and transition layers (Part II)**, G. Wheeler, P. Rybka

Glen Wheeler

James McCoy

Simon Blatt

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**Contributed session (Part VI)**

Enrico Valdinoci

Subas Acharya

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Yvonne Stokes

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12:45–14:45: Parallel Sessions

**Free boundary problems in cell biology (Part II)**, C. Gräser, M. Röger

Chandrasekhar Venkataraman

Philip Herbert

Diane Peurichard

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**Surface PDEs (Part II)**, C. Venkataraman, T. Ranner

Axel Voigt

Paola Pozzi

Balázs Kovács

Room 2

Matthias Röger

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**Asymptotic approaches to interface dynamics (Part II)**, J. José López Velázquez, J. King

Jonas Jansen

Mark Blyth

Linda Cummings

Room 3

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15:00–15:45: Plenary

**The size of the singular set in the Stefan problem**

Xavier Ros-Oton

Lecture hall

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15:45–16:00: Closing

Lecture hall

## Plenary Talks

## Plenary Talk

Mon 09:00–09:45

**Antonin Chambolle**, CNRS and CEREMADE, Université Paris-Dauphine, PSL, France

**Discretizations of the total variation for singular functions**

The total variation, which is used as a regularizer in inverse problems in imaging or to compute minimal surfaces, is the simplest example of a convex functional whose minimizers are typically discontinuous. This raises interesting numerical issues, as there are many ways to discretize it, most of which with drawbacks such as a loss of isotropy or important numerical diffusion. In this talk I will review some recent results (obtained in collaboration with C. Caillaud and T. Pock (TU Graz)) about the discretization errors for this functional, and introduce a family of consistent discretizations which can be improved by a learning strategy, yielding sharper interfaces or “nicer” image denoising.

## Plenary Talk

Mon 15:00–15:45

**Irene Fonseca**, Carnegie Mellon University

**Phase transitions in heterogeneous media: equilibria and geometric flows**

A variational model in the context of the gradient theory for fluid-fluid phase transitions with small scale heterogeneities is studied. In the case where the scale of the small heterogeneities is of the same order of the scale governing the phase transition, the interaction between homogenization and the phase transitions process leads to an anisotropic interfacial energy. The underlying gradient flow provides unconditional convergence results for an Allen-Cahn type bi-stable reaction diffusion equation in a periodic medium. curvature, is obtained. This is joint work with Riccardo Cristoferi (Radboud University, The Netherlands), Adrian Hagerty, Cristina Popovici, Rustum Choksi (McGill), Jessica Lin (McGill), and Raghavendra Venkatraman (CMU).

## Plenary Talk

Mon 16:15–17:00

**Robert Saye**, Lawrence Berkeley National Laboratory  
**Higher-Order Numerics for Interfacial Fluid Dynamics**

Surface, boundary, and interface motion play a pivotal role in a wide range of fluid dynamics problems. When modelling these problems computationally, a careful and precise treatment of the interface is often necessary - boundary layers can strikingly affect interface motion, while small-scale features in surface geometry can affect fluid dynamics far afield. In this talk, we discuss some of our work on developing higher-order numerical methods for interfacial fluid dynamics, including implicit mesh discontinuous Galerkin (DG) methods which utilize level set methods to create implicitly-defined meshes capable of imposing or capturing interfacial jump conditions with high-order accuracy. Highlighted topics include high-order quadrature algorithms for multi-component implicitly defined geometry, and fast multigrid solvers for DG discretizations of elliptic interface/Stokes problems. We also present some applications, including: high-Reynolds number soap bubble oscillations; multi-scale models of membrane rearrangement, drainage and rupture in foam dynamics; vortex shedding dynamics created by intricate Plateau-Rayleigh instabilities; and modeling rotary bell atomization in industrial spray painting processes.

## Plenary Talk

Tue 09:00–09:45

**Yoshihiro Tonegawa**, Tokyo Institute of Technology  
**Generalized mean curvature flow**

I will discuss a time-global existence theorem of generalized mean curvature flow which can move with singularities such as triple junctions and which goes through various topological changes as it evolves. It is a so-called Brakke flow but it additionally satisfies other characterizations such as BV characterization. I plan to give a sketch on how to construct the solution and what it satisfies.

Plenary Talk

Tue 15:00–15:45

Lia Bronsard, McMaster University

**Patterns in tri-block copolymers: droplets, double-bubbles and core-shells**

We study the Nakazawa-Ohta ternary inhibitory system, which describes domain morphologies in a triblock copolymer as a nonlocal isoperimetric problem for three interacting phase domains. The free energy consists of two parts: the local interface energy measures the total perimeter of the phase boundaries, while a longer-range Coulomb interaction energy reflects the connectivity of the polymer chains and promotes splitting into micro-domains. We consider global minimizers on the two-dimensional torus, in a limit in which two of the species have vanishingly small mass but the interaction strength is correspondingly large. In this limit there is splitting of the masses, and each vanishing component rescales to a minimizer of an isoperimetric problem for clusters in 2D. Depending on the relative strengths of the coefficients of the interaction terms we may see different structures for the global minimizers, ranging from a lattice of isolated simple droplets of each minority species to double-bubbles or core-shells. This represents work with S. Alama, X. Lu, and C. Wang.

Plenary Talk

Wed 09:00–09:45

Sören Bartels, University of Freiburg

**Simulation of Free Boundary Problems in the Non-linear Bending of Elastic Rods and Plates**

The mathematical description of large bending deformations of thin elastic rods and plates leads to fourth order problems with nonlinear pointwise constraints that give rise to various free boundary phenomena. The free boundary may separate regions of trivial and large deformations, describe the contact zone in the presence of an obstacle, or be related to the occurrence of self-contact. We devise and analyse numerical methods that are capable of reliably capturing these effects under minimal regularity assumptions and which allow us to experimentally study topological transitions of free boundaries.

Plenary Talk

Wed 15:00–15:45

Qiang Du, Columbia University

**Nonlocal variational geometric problems with free boundary**

We will discuss a couple of variational problems in geometry with free boundary. These problems are motivated from applications in biology and chemistry and all involve nonlocal contributions in the energy functionals. We will study both analytical and numerical aspects of the variational problems and present algorithms that can be used to explore the energy landscape.

Plenary Talk

Thu 09:00–09:45

Barbara Niethammer, University of Bonn

**An obstacle problem for cell polarization**

We investigate a model for cell polarization under external stimulus where a diffusion equation in the inner cell is coupled to reaction diffusion equations on the cell membrane. In certain scaling limits we rigorously derive generalized obstacle type problems. For these limit systems we prove global stability of steady states and characterize the parameter regime for the onset of polarization.

Plenary Talk

Thu 15:00–15:45

Alessio Figalli, ETH-Bereich Hochschulen  
**Generic regularity in obstacle problems**

The classical obstacle problem consists of finding the equilibrium position of an elastic membrane whose boundary is held fixed and which is constrained to lie above a given obstacle. By classical results of Caffarelli, the free boundary is  $C^\infty$  outside a set of singular points. Explicit examples show that the singular set could be in general  $(n - 1)$ -dimensional — that is, as large as the regular set. In a recent paper with Ros-Oton and Serra we show that, generically, the singular set has zero  $\mathcal{H}^{n-4}$  measure (in particular, it has codimension 3 inside the free boundary), solving a conjecture of Schaeffer in dimension  $n \leq 4$ . The aim of this talk is to give an overview of these results.

Plenary Talk

Thu 18:20–19:05

Andrea Bertozzi, UCLA math department  
**Energy Minimizing Surface Tension Configurations for Microparticles**

Drop-Carrier Particles (DCPs) are solid microparticles designed to capture uniform microscale drops of a target solution without using costly microfluidic equipment and techniques. DCPs are useful for automated and high-throughput biological assays and reactions, as well as single cell analyses. DCPs enable templated uniform-sized droplets due to energy minima achieved for the free boundary problem of the interface between immiscible liquids such as oil and water. Each DCP has an energy-volume graph a finite volume achieving the lowest energy state. This allows for multiple DCPs to exchange fluid so that a system of particles has a lowest energy state in which all but one DCP contain the same uniform volume. We compare the theoretical prediction for the volume distribution to macroscale experiments of pairwise droplet splitting, with good agreement. This leads to a statistical theory for the number of pairwise interactions of DCPs needed to reach a uniform volume distribution. Heterogeneous mixtures of DCPs with different sized particles require fewer interactions to reach a minimum energy distribution for the system. We present ideas for the optimization of the DCP geometry for minimal required target solution and uniformity in droplet volume.



## Plenary Talk

Fri 09:00–09:45

Enrique Zuazua, Friedrich-Alexander-Universität Erlangen-Nürnberg

### Inverse design for conservation laws and Hamilton-Jacobi equations

We discuss the problem of inverse design, or time inversion, for scalar conservation laws and Hamilton-Jacobi equations. The presence of singularities in the forward evolution is an obstruction for backward uniqueness and the unilateral bounds generated by the forward dynamics establish thresholds on the set of reachable data. In this lecture we shall present our recent works in collaboration with Thibault Liard and Carlos Esteve, characterising the set of reachable states, identifying the multiplicity of initial data leading to a final target, and determining the role of backward entropy or viscous solutions. We also develop numerical algorithms allowing to reconstruct the set of inversions and present some numerical experiments.

## Plenary Talk

Fri 15:00–15:45

Xavier Ros-Oton, ICREA & Universitat de Barcelona

### The size of the singular set in the Stefan problem

The Stefan problem, dating back to the XIXth century, is probably the most classical and important free boundary problem. The regularity of free boundaries in the Stefan problem was developed in the groundbreaking paper (Caffarelli, Acta Math. 1977). The main result therein establishes that the free boundary is  $C^\infty$  in space and time, outside a certain set of singular points. The fine understanding of singularities is of central importance in a number of areas related to nonlinear PDEs and Geometric Analysis. In particular, a major question in such context is to establish estimates for the size of the singular set. The goal of this talk is to present some new results in this direction for the Stefan problem in  $\mathbb{R}^3$ . This is a joint work with A. Figalli and J. Serra.

Mon 09:50–11:50

## Minisymposium

Mon 09:50–11:50 Room 1

**Interfaces in fluids (Part I)**

Organizers: H. Abels, H. Garcke, M. Wilke

Christina Lienstromberg, University of Bonn

**Analysis of non-Newtonian thin-film equations**

I will offer an insight into mathematical models describing the dynamic behavior of non-Newtonian thin films. The resulting PDEs are in general nonlinear, degenerate, of fourth order, and with a possibly “weak” dependence of the coefficients. I will discuss recent results on such an evolution equation for the interface separating two viscous immiscible fluids, confined between two concentric cylinders rotating at a small relative velocity. In this so-called Taylor-Couette setting, two competing effects drive the dynamics of the interface - the surface tension and the shear stresses induced by the rotation of the cylinders. When the two effects are comparable, solutions behave, for large times, as in the Newtonian regime. For the regime in which surface tension effects dominate the stresses induced by the rotating cylinders, we prove local existence of positive weak solutions for both shear-thinning and shear-thickening fluids. In the case of a shear-thickening fluid, one observes that interfaces which are initially close to a circle converge to a circle in finite time. The talk is based on a joint work with Tania Pernas-Castaño and Juan Velázquez: <https://arxiv.org/abs/2012.10734>.

Elisabetta Rocca, University of Pavia

**A phase-field prostate cancer growth with chemotherapy and antiangiogenic therapy effects**

Here, we present a mathematical model of prostate cancer growth and chemotherapy recently introduced in collaboration with P. Colli, H. Gomez, G. Lorenzo, G. Marinoschi, an A. Reali. We use the phase-field method to describe tumor growth, which we assume to be driven by a generic nutrient following reaction-diffusion dynamics. Cytotoxic chemotherapy is included as a term downregulating tumor net proliferation, while antiangiogenic therapy is modeled as a reduction in intratumoral nutrient supply. An additional equation couples the tumor phase field with the production of prostate-specific antigen. We prove the well-posedness of our model and we run a series of simulations leveraging an isogeometric method to explore the effects of cytotoxic and antiangiogenic therapies.

Piotr B. Mucha, University of Warsaw

**Free boundary problems in viewpoint of DaPrato-Grisvard theorem**

I plan to talk about an approach to the free boundary problems in the special functional setting. We aim at showing stability in time, hence generally we are required to have information in the  $L^1$  in time. Thanks to the abstract approach based on the classical DaPrato-Grisvard theorem, we are able to reproduce the estimate for a linearization in the  $L^1(0, \infty; X)$  spaces, here  $X$  is a real interpolation space given in terms of Besov framework. The talk is based on joint result with R. Danchin, M. Hieber, P. Tolksdorf.

Marius Tucsnak, University of Bordeaux

**Large time behavior in some fluid-interaction problems**

We consider several systems modelling the motion of rigid bodies in a viscous fluid. We discuss the long time behavior of solutions in terms of the fluid properties (incompressible or compressible) and on the geometric setting (bounded or unbounded domains in one or three space dimensions). The focus is on the adiabatic piston problem in one space dimension

(compressible case) and on the large time behavior of a solid moving in an unbounded three dimensional incompressible viscous fluid.

## Minisymposium

Mon 09:50–11:50 Room 2

**Optimization and control of FBPs (Part I)**

Organizers: M. Hintermüller, M. Hinze

Christian Kahle, Universität Koblenz-Landau

**Optimal control of sliding droplets using the contact angle**

We present results on optimal control of sliding droplets. Here the contact angle between droplet and solid serves as a control variable. The fluid is modeled by a thermodynamically consistent diffuse interface model with a suitable contact line model. In earlier work [H. Bonart, C. Kahle, J.-U. Repke, JCP 399 (2019)] we compared different time discretization schemes for this model that mimics the energy behaviour of the continuous model. We now employ a particular scheme to derive existence of optimal controls for a time discrete optimal control problem and also first order necessary conditions. As controls we consider finite dimensional controls for the contact angle distribution. We test our approach by driving a droplet up an inclined plate.

Tobias Keil, Weierstrass Institute

**Optimal control of a coupled Cahn–Hilliard–Navier–Stokes system with variable fluid densities**

This talk is concerned with the optimal control of two immiscible fluids. For the mathematical formulation a coupled Cahn–Hilliard–Navier–Stokes system is used which involves a variational inequality of 4th order. We discuss the differentiability properties of the control-to-state operator and the corresponding stationarity concepts for the control problem. Strong stationarity conditions are presented and a numerical solution algorithm based on a bundle-free implicit programming approach is provided which terminates at an at least C-stationary point which, in the best case, is even strongly stationary.

Kei Fong Lam, Hong Kong Baptist University

**Consistency of a phase field regularisation for an inverse problem governed by a quasilinear Maxwell system**

We tackle an inverse problem of reconstructing a discontinuous coefficient in quasilinear  $H(\text{curl})$  magnetostatic equations from measurements in a subdomain. To overcome the ill-posedness of the inverse problem, we investigate two regularisations posed as constrained minimisation problems. The first involves perimeter penalisation and the second involves phase field regularisation. Existence of minimisers and consistency as the penalisation parameters tending to zero are discussed. We show under ideal situations a relation between parameters that allows one to obtain a solution to the inverse problem from the phase field solutions, and then we investigate the sharp interface limit of the first order optimality conditions of the phase field problem. This is joint work with Irwin Yousept.

## Minisymposium

Mon 09:50–11:50 Room 3

**Numerical methods for surface-bulk problems (Part I)**

Organizers: A. Reusken, E. Bänsch

Markus Gahn, University of Heidelberg - Interdisciplinary Center for Scientific Computing (IWR)

**A mixed finite element method for coupled bulk-surface problems including Wentzell-boundary conditions**

In this talk we introduce and analyze a mixed finite-element approach for an elliptic coupled bulk-surface problem with Wentzell-boundary condition of second order. Such kind of models can be obtained by dimension reduction and play an important role in many applications for example in the medical sciences. The model is formulated on a domain with a curved smooth boundary and we introduce a mixed formulation that is equivalent to the usual weak formulation. Furthermore, we derive optimal a priori error estimates between the exact solution and the finite-element approximation. To this end the curved domain is approximated by a polyhedral domain introducing an additional geometrical error that has to be bounded.

André Massing, Norwegian University of Science and Technology

**High order cut finite element methods for coupled bulk surface problems [canceled]**

In this talk we present a unified framework for the formulation of unfitted finite element methods for elliptic and hyperbolic problems. Both continuous and discontinuous Galerkin methods are considered. The resulting framework allows for the unified numerical treatment of a wide range of problems, including boundary and interface problems as well as surface and mixed-dimensional, coupled surface-bulk and interface-bulk problems. In the second part of the talk, we discuss how to combine the framework with high-order time-stepping methods to discretize coupled surface-bulk problems on moving domains.

Dirk Peschka, Weierstrass-Institut für Angewandte Analysis und Stochastik

**Dynamic contact and their numerical discretization for gradient systems**

In this talk we will discuss an energy-based framework for free boundary problems with dynamic contact angle and investigate the numerical discretization of such nonlinear PDE problems. Firstly, the mathematical structure of the gradient flow and the resulting weak formulation of the problem will be described. Then, we present the time discretization of the PDE system. Finally, different relevant applications and solutions will be presented.

Mon 12:45–14:45

## Minisymposium

Mon 12:45–14:45 Room 1

**Free boundary problems related to shapes and geometries**

Organizers: E. Oudet, G. Buttazzo

Giuseppe Buttazzo, University of Pisa

**Upper and lower bounds for some shape functionals**

The relations between two quantities related to the Laplace operator are considered. In particular, taking as a model the heat diffusion, governed by the heat equation, we aim to study the relations between the average temperature of a heated body and the temperature decay rate of the body in absence of heat sources. The quantities above are expressed by the so-called “torsional rigidity” and by the principal eigenvalue of the Laplace operator. The relations above are studied in the classes of general domains, convex domains, and domains with a small thickness. This allows to obtain a detailed description of the Blasche-Santaló diagram of the two quantities. Several open questions are discussed, in particular when the Laplacian is replaced by the  $p$ -Laplacian.

Edouard Oudet, Université Grenoble Alpes

**Metric Optimization in Spectral geometry**

The results of this talk on recent numerical aspects of metric optimization have been obtained in collaboration with Chiu-Yen Kao and Braxton Osting. The first part of the talk is dedicated to Nash’s isometric embedding theorem for surfaces. We recall the impressive results obtained by HEVEA’s project for the flat torus (V. Borrelli, F. Lazarus, B. Thibert et al.) and illustrate how spectral formulation may lead to a new intrinsic approach which are not related to Gromov’s construction. Following the theoretical results of Fraser and Schoen, we describe in a second part a numerical approach to approximate minimal surfaces in the ball that is surfaces (i) contained in the ball (ii) that have zero mean curvature and (iii) meet the boundary of the ball orthogonally.

Bozhidar Velichkov, University of Naples Federico II

**Regularity of the optimal sets for the second eigenvalue of the Dirichlet Laplacian**

We prove a regularity theorem for the sets  $\Omega$  that minimise the functional  $\lambda_2(\Omega) + |\Omega|$  among all open sets contained in a given smooth domain  $D \subset \mathbb{R}^d$ ;  $\lambda_2(\Omega)$  being the second eigenvalue of the Laplacian with Dirichlet boundary conditions on  $\partial\Omega$ , and  $|\Omega|$  the Lebesgue measure of  $\Omega$ . Precisely, we prove that  $\Omega$  is equivalent to the disjoint union  $A \cup B$ , where the boundaries of the sets  $A$  and  $B$  are  $C^{1,\alpha}$  regular manifolds up to a closed singular set, which is empty in dimension  $d \leq 4$  and has Hausdorff dimension at most  $d - 5$ , when  $d \geq 5$ . This is a joint work with Dario Mazzoleni (University of Brescia) and Baptiste Trey (University of Grenoble Alpes).

## Minisymposium

Mon 12:45–14:45 Room 2

**Phase field models (Part I)**

Organizers: E. Rocca, V. Styles

Klaus Deckelnick, Otto-von-Guericke-Universität Magdeburg

**A practical phase field method for an elliptic surface PDE**

We consider a diffuse interface approach for solving an elliptic PDE on a given closed hypersurface. The method is based on a (bulk) finite element scheme employing numerical quadrature for the phase field function and hence is very easy to implement compared to other approaches. We estimate the error in natural norms in terms of the spatial grid size, the interface width and the order of the underlying quadrature rule. Numerical test calculations are presented which confirm the form of the error bounds. This is joint work with John W. Barrett and Vanessa Styles.

Pierluigi Colli, University of Pavia

**Optimal control problems with deep quench approach and sparsity for phase field tumor growth models**

The talk reports on some recent results obtained in collaboration with Andrea Signori and Juergen Sprekels. A distributed optimal control problem is considered for a tumor growth model of Cahn–Hilliard type including chemotaxis. The evolution of the tumor fraction is governed by a pointwise inclusion involving the subdifferential of a double obstacle potential. The control and state variables are nonlinearly coupled and the cost functional contains a nondifferentiable term like the  $L^1$ -norm in order to include sparsity effects. To face the difficulties in the analysis, the so-called “deep quench approach” is employed so that the convex part of the double obstacle potential is approximated by logarithmic functions from the interior of the domain. This approximation is used to derive first-order optimality conditions also for the double obstacle case, by obtaining a variational inequality in terms of the associated adjoint state variables. Finally, if time permits sparsity results for the optimal controls are discussed.

Michael Hinze, Universität Koblenz-Landau

**Shape and topology optimization in fluids using phase field models**

We consider a phase field approach to shape and topology optimization in fluid flow. The mathematical modeling leads to a PDE constrained optimization problem with control in the coefficients where the control enters as phase field in the Darcy term of the Navier–Stokes model. We prove existence of solutions and present a numerical realization based on the finite element method. We illustrate the performance of our approach with some numerical examples.

Maurizio Grasselli, Politecnico di Milano

**Conserved Allen-Cahn-Navier–Stokes systems**

I intend to present some results on a model of phase separation in (incompressible) liquids proposed by M.-H. Giga, A. Kirshtein, and C. Liu in 2018. This model consists of the Navier–Stokes equations coupled with the Allen-Cahn equation and accounts for a variable density which may depend on the phase field. I shall focus on the conserved Allen-Cahn equation with logarithmic potential. These results have been obtained jointly with A. Giorgini (Indiana University, USA) and H. Wu (Fudan University, PRC).

Tue 09:50–11:50

Minisymposium

Tue 09:50–11:50 Room 1

**Geometric problems (Part I)**

Organizers: M. Novaga, Y. Giga

Miyuki Koiso, Kyushu University

**Stable anisotropic capillary hypersurfaces in a wedge and application to partially-crystalline variational problems**

We study surfaces with constant anisotropic mean curvature in a wedge and with free boundary on the boundary of the wedge in the three-dimensional euclidean space, which are critical points of “the anisotropic surface energy and the wetting energy” for variations preserving the enclosed volume. We show a uniqueness result for local minimizers of the total energy. The results are generalized to hypersurfaces in higher dimensional spaces. Moreover, the method is applied to the uniqueness problem for stable solutions of partially-crystalline variational problems for piecewise smooth hypersurfaces.

Tatsuya Miura, Tokyo Institute of Technology

**Variational analysis of self-intersecting elastic curves**

In this talk we study variational problems involving bending energy for closed curves in Euclidean space under constraints on self-intersections. We obtain optimal configurations, some of which are nonclassical shapes, and discuss applications and connections to elastic flows, elastic networks, and elastic knots.

Enrico Valdinoci, University of Western Australia

**(Non)local phase coexistence models**

We will discuss classical and recent results concerning the Allen-Cahn equation and its long-range counterpart, especially in relation to its limit interfaces provided by (possibly nonlocal) minimal surfaces.

Minisymposium

Tue 09:50–11:50 Room 2

**Interfaces in fluids (Part II)**

Organizers: H. Abels, H. Garcke, M. Wilke

Dieter Bothe, TU Darmstadt

**On the velocity jump discontinuity for bubbles rising in a viscoelastic fluid**

It is well-known that bubbles rising in viscoelastic liquids may exhibit a jump discontinuity of the rise velocity as a critical bubble volume is exceeded. This phenomenon has been extensively investigated in the literature, both by means of experiments as well as numerical simulations. The occurrence of the velocity jump seems to be associated with a change of the bubble shape under formation of a pointed tip at the rear and to the appearance of a so-called negative wake with the liquid velocity behind the bubble pointing in the opposite direction to that in viscous Newtonian fluids. We revisit this topic, reporting in particular our findings from a local conformation tensor analysis, which is able to explain a true discontinuity in the terminal bubble rise velocity. This is joint work with G. Brenn (TU Graz) and M. Niethammer (TU Darmstadt).

Philippe Laurençot, University of Toulouse

**Some Properties of the thin film Muskat problem**

The thin film Muskat describes the space-time evolution of the heights of two fluid layers with different viscosities and densities and is a second-order degenerate parabolic system featuring a full diffusion matrix. Some properties of this system will be reviewed, mostly in one space dimension, including existence of non-negative weak solutions, finite speed of propagation, time-independent estimates, and large time behavior. The connection with the porous medium equation will be discussed as well. Joint work with Bogdan-Vasile Matioc (Regensburg).

Hans Knüpfer, Universität Heidelberg

**Self-similar lifting and persistent touch-down point solutions in the thin-film equation**

In the talk I discuss the appearance of self-similar blow-up solutions for thin-film equations with different mobility exponents. This is related to non-uniqueness phenomena for weak solution of the same equation. The proof is based on dynamical systems arguments.

Hao Wu, Fudan University

**Analysis of a diffuse interface model for two-phase incompressible flow in porous medium [canceled]**

We study a diffuse interface model for incompressible two-phase flows in a porous medium. Existence of global weak solutions in the three dimensional case is established by using the monotone operator method. In the two-dimensional case, we prove the existence and uniqueness of global strong solutions. Finally, we show that weak solutions of our system will converge to the weak solution of the Cahn–Hilliard–Darcy system as the viscosity parameter vanishes.

## Minisymposium

Tue 09:50–11:50 Room 3

**Free boundary problems for nonlinear hyperbolic/mixed-type PDEs (Part I)**

Organizer: G.-Q. Chen

Myoungjean Bae, KAIST

**Recent Progress on the Study of Euler-Poisson system**

In this talk, I will present recent results on three types (subsonic, supersonic, smooth transonic) of solutions to the steady Euler-Poisson system

$$\begin{cases} \operatorname{div}_x(\rho \mathbf{u}) = 0 \\ \operatorname{div}_x(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla_x p = \rho \nabla_x \phi \\ \operatorname{div}_x(\rho \mathbf{u} \mathbf{B}) = \rho \mathbf{u} \cdot \nabla_x \Phi \\ \Delta_x \Phi = \rho - \rho_I \end{cases}$$

This talk is based on several joint works with B. Duan (Dalian University of Technology), H. Park (Yonsei University), J.-J. Xiao (Chinese University of Hong Kong) and C. Xie (Shanghai Jiao Tong University)

Hyangdong Park, Yonsei University

**Contact discontinuities for inviscid compressible flows**

We prove the existence of a subsonic axisymmetric weak solution to steady Euler system in a three-dimensional infinitely long cylinder when prescribing the values of the entropy and angular momentum density at the entrance by piecewise  $C^2$  functions with a discontinuity on a curve on the entrance. Due to the variable entropy and angular momentum density conditions with a discontinuity at the entrance, the corresponding solution has a nonzero vorticity, nonzero angular momentum density, and contains a contact discontinuity. We construct such a solution via Helmholtz decomposition. (This is a joint work with Myoungjean Bae.)

Tao Wang, Wuhan University

**Free Boundary Problems in Ideal Compressible MHD**

I will present the joint works with Professor Yuri Trakhinin on the local well-posedness of free boundary problems in ideal compressible MHD with or without surface tension. The Rayleigh-Taylor sign condition on the total pressure is required only for the case of zero surface tension.

Wei Xiang, City University of Hong Kong

**Stability of contact discontinuity in nozzles**

There can be contact discontinuities in the nozzle for the compressible Euler flow. In this talk, we will talk about the cases of the subsonic-subsonic, subsonic-supersonic, and supersonic-supersonic contact discontinuity in nozzles and establish the well-posedness results.

Tue 12:45–14:45

Minisymposium

Tue 12:45–14:45 Room 1

**Phase field models (Part II)**

Organizers: E. Rocca, V. Styles

Luca Scarpa, Politecnico di Milano

**On a class of nonlocal phase-field models for tumour growth**

We present some recent results on a class of phase-field models for tumour growth, coupling a nonlocal viscous Cahn–Hilliard equation for the phase-variable (local concentration of tumoral cells) and a reaction-diffusion equation for the nutrient concentration (e.g. glucose). The presence of chemotaxis and active transport terms is also accounted for. We first discuss weak/strong well-posedness of the system, separation properties for the phase-variable in case of singular potentials, and vanishing-viscosity asymptotics. Secondly, we investigate some selected optimal control problems associated to optimality of treatment or parameter identification. This study is based on joint works with Elisabetta Rocca and Andrea Signori.

Björn Stinner, University of Warwick

**Automatic phase field regularisation of moving boundary problems**

We present a software interface that enables users to specify moving boundary problems that then are transformed into a phase field model, which subsequently can be solved by binding to efficient finite element software backends. Many moving boundary problems amenable to a phase field approximation have a common structure, namely, equations of gradient flow type of an interfacial energy are coupled with some balance laws. We discuss a preliminary classification of such problems and, with examples, illustrate how the software symbolically manipulates the input parameters into the variational formulation of a phase field model. We also address variants of regularisation and time-discretisation, and the specification of the spatial discretisation including adaptivity and numerical solvers.

Marita Thomas, Weierstrass Institute for Applied Analysis and Stochastics

**Convergence analysis for fully discretized damage and phase-field fracture models**

This presentation deals with techniques for the spatial and temporal discretization of models for rate-independent damage and phase-field fracture featuring a gradient regularization and a non-smooth constraint due to the unidirectionality of the damage process. Both quasi-static and dynamic evolution of the displacements is addressed. Suitable notions of solution for the non-smooth process are introduced and the corresponding discrete version is studied by combining a time-discrete scheme with finite element discretizations of the domain. Results and challenges on the convergence of the discrete problems in the sense of evolutionary Gamma-convergence in dependence of the choice of the time-discrete scheme, the gradient term and the mesh properties are discussed.

Harald Garcke, Universität Regensburg

**Cahn–Hilliard–Brinkman models for tumour growth**

Tumour growth models have been successful in describing many phenomena relevant for medical applications. We will introduce phase field systems for tumour growth by coupling the Cahn–Hilliard equation to a diffusion equation for a nutrient. In addition, also a coupling to flow equations of Darcy-, Stokes- and Brinkman-type are discussed. We will present existence and uniqueness results, study sharp interface limits and briefly

discuss patient specific parameter estimation using reduced order modeling.

## Minisymposium

Tue 12:45–14:45 Room 2

**UQ in free boundary problems**

Organizers: H. Harbrecht, M. D. Multerer

Caroline Geiersbach, Weierstrass Institute

**Stochastic Approximation for Optimization in Shape Spaces**

In this talk, we present a novel approach to solve stochastic shape optimization problems. Our approach is the extension of the classical stochastic gradient method, dating back to Robbins and Monro (1951), to infinite-dimensional shape manifolds. The basic paradigm of stochastic approximation is the use of partial function information combined with decreasing step sizes. We prove convergence of the method on Riemannian manifolds and make the connection to shape spaces. The method is demonstrated on an interface identification problem, where uncertainty arises in the form of a random partial differential equation. We verify conditions for convergence for the model problem and demonstrate the method numerically. (Joint work with Kathrin Welker and Estefania Loayza-Romero.)

Alexey Chernov, University of Oldenburg

**Numerical solution of rough random obstacle problems**

The obstacle problem is a prominent free boundary problem that consists of finding the profile of an elastic membrane in equilibrium under the constraint that it is located above a given obstacle function. We are particularly interested characterizing solutions for a class of rough randomly perturbed obstacles that may model rough manufactured materials (e.g. asphalt road surface) or rough biological microstructures. Importantly, due to randomly changing location of the free boundary, the parameter-to-solution map has only limited global regularity that makes various polynomial-based approximation methods of Numerical Uncertainty Quantification notoriously inefficient. We propose and analyse a family of Multilevel Monte Carlo algorithms that do not suffer from this lack of regularity.

Nick Wulbusch, University of Oldenburg

**Shape optimization algorithms for elliptic problems under uncertainty**

We consider a stochastic PDE constrained shape optimization problem. The constraint is represented by a stationary diffusion problem having a source term with an uncertain location. A typical goal functional represents the misfit between the computed solution and a prescribed function within the computational domain. The aim is to determine the optimal shape of the domain boundary minimizing the goal functional. We investigate theoretically and numerically convergence of gradient-based optimization algorithms and discuss possible extensions to the time-harmonic acoustic problems in the low frequency regime.

Helmut Harbrecht, University of Basel

**Solving a Bernoulli type free boundary problem with random diffusion**

The present talk is concerned with the numerical solution of a free boundary problem for an elliptic state equation with random diffusion. The domain under consideration is represented by a level set function which is evolved by the objective's shape gradient. The state is computed by the finite element method, where the underlying triangulation is constructed by means of a marching cubes algorithm. The high-dimensional integral, which is induced by the random diffusion, is approximated by the quasi-Monte Carlo method. By numerical experiments, we validate the feasibility of the approach.



Tue 17:05–19:05

Minisymposium

Tue 17:05–19:05 Room 1

**Numerical methods for geometric PDEs (Part I)**

Organizers: R. Nochetto, R. Nürnberg

Andrea Bonito, Texas A&amp;M University

**Numerical Approximations of Plates Undergoing Large Deformations and Folding**

Thermostats, microgrippers, sensors, telescopes, self-deployable structures, growth in soft tissues, growth in leaves or fungus, nano actuators, flapping and flytrap mechanisms, and origami are diverse examples among many others where a plate-like device undergo large deformations. With these applications in mind, we consider thin sheets endowed with different elastic energies modeling either lattice mismatches between two layers of polymers, internal stresses, or defects in the materials. Their common feature is that they result in forth order problems for the plate deformation. However, the possibility of large deformations rules out standard linear models. Instead, the first fundamental forms of the plate deformations are constrained to the identity, if the plate cannot sustain shear nor stretch, or more generally to a given prestrain metric. The numerical algorithms for the approximation of the plate deformations are based on local discontinuous Galerkin methods, where high order derivatives in the continuous models are replaced by weakly converging discrete reconstructions. We briefly describe the models, algorithms along with their analysis, and explore numerically their capabilities.

Wenbo Li, University of Tennessee, Knoxville

**Finite element discretizations of nonlocal minimal graphs**

We study the discretizations of nonlocal minimal graphs of order  $s \in (0, 1/2)$ , minimizing the so-called fractional perimeters. Such a Plateau problem can be reinterpreted as a Dirichlet problem for a nonlocal, nonlinear, degenerate operator of order  $s + 1/2$ . We propose a numerical method based on piecewise linear Lagrange finite elements on shape-regular meshes, and prove existence, uniqueness and convergence of discrete solutions. In addition, We derive error estimates for a novel geometric quantity related to the concept of fractional normal. We also present numerical experiments that illustrate the performance of our scheme and behaviors of nonlocal minimal graphs including the stickiness phenomenon. This is a joint work with Juan Pablo Borthagaray and Ricardo H. Nochetto.

Shawn Walker, Louisiana State University

**A Mixed FEM for the Plate Equation on Surfaces: the Surface HHJ Method**

We present a mixed finite element method for solving a fourth order elliptic PDE, the Kirchhoff plate equation, on a surface embedded in  $\mathbb{R}^3$ , with or without boundary. The method discretizes the surface Hessian directly, is built on the classic Hellan–Herrmann–Johnson (HHJ) method (for flat domains), and convergence is established for a  $C^{k+1}$  surface, with  $C^0$ , degree  $k$  (Lagrangian) approximation of the surface, for any  $k \geq 1$ . Mixed boundary conditions are allowed, including clamped, simply-supported, and free conditions. Numerical examples are presented that demonstrate the method, the use of “point” boundary conditions, and how to solve the surface biharmonic equation, which has many applications, such as surface (thin film) flows and surface diffusion.

Sören Bartels, University of Freiburg

**Local Mesh Refinement in the Numerical Approximation of TV Regularized Variational Problems**

Recent quasi-optimal error estimates by Chambolle and Pock for the finite element approximation of total-variation regularized minimization problems require the existence of a Lipschitz continuous dual solution. We discuss the validity of this condition and devise numerical methods using locally refined meshes that lead to improved convergence rates despite the occurrence of discontinuities. It turns out that linear convergence is possible on suitably constructed meshes. This is joint work with Friedrich Wassmer (University of Freiburg).

## Contributed Session

Tue 17:05–19:05 Room 2

## Contributed session (Part I)

Diogo Caetano, University of Warwick

**The Cahn–Hilliard equation on an evolving surface**

We describe a setting for posing evolution PDEs on evolving hypersurfaces which is suitable for formulating the Cahn–Hilliard equation on a moving surface. We consider well-posedness for smooth, logarithmic, and double obstacle nonlinearities and with a constant mobility term. It turns out that for the singular potentials some conditions relating the initial data and the evolution of the surfaces are necessary for (global in time) existence of solutions, and we identify these different regimes. We then consider a related model that does not preserve the integral of solutions, and establish existence results. Time permitting, we will also discuss some open questions and related problems. This is joint work with Charlie Elliott (U. of Warwick).

Lars von Wolff, University of Stuttgart

**A Ternary Cahn–Hilliard Navier–Stokes Model for Two Phase Flow with Precipitation and Dissolution**

Multiphase flow and reactive transport are important in many applications, for example in porous media. We consider the incompressible flow of two immiscible fluids in the presence of a solid phase changing due to precipitation and dissolution of ions. We propose a novel phase-field model that extends the widespread two-phase models to the ternary case by including a solid phase. Here the fluid-solid interaction poses new difficulties. The model obeys the second law of thermodynamics for a free energy including the ion concentration. We analyze the sharp interface limit, in particular for the three-phase contact point. Notably, the new phase-field model realizes Navier-slip conditions for solid-fluid interfaces. We conclude with a comparison of numerical simulations with experimental data.

Alice Marveggio, Institute of Science and Technology Austria (IST)

**On a non-isothermal Cahn–Hilliard model based on a microforce balance**

We consider a non-isothermal Cahn–Hilliard model based on a microforce balance. The model was derived by A. Miranville and G. Schimperna starting from the two fundamental laws of Thermodynamics, following M. Gurtin's two-scale approach. The main working assumptions are on the choice of the Ginzburg-Landau free energy, and on the behaviour of the heat flux as the absolute temperature tends to zero and to infinity. By deriving suitable a priori estimates and by showing weak sequential stability of families of approximating solutions, we prove global-in-time existence for the initial-boundary value problem associated to the entropy formulation and, in a sub-case, also to the weak formulation of the model. (Joint work with G. Schimperna)

Makoto Okumura, Hokkaido University

**The solvability of a discrete variational derivative scheme for the equation and boundary condition of the Cahn–Hilliard type**

In this talk, we propose a structure-preserving scheme for the Cahn–Hilliard equation with dynamic boundary conditions introduced by Goldstein et al. (2011). In this model, the total mass conservation and the total energy dissipation hold. We design a structure-preserving scheme for the target equation so that the scheme retains the properties using the discrete variational derivative method proposed by Furihata-Matsuo (2010). Then, how to discretize the energy which characterizes the equation,

it is important and essential. Indeed, modifying the conventional manner and using an appropriate summation-by-parts formula, we improve the structure-preserving scheme for the target problem. Also, we prove the solvability of the proposed scheme and show the results of numerical experiments.

Tue 17:05–19:05 Room 3

Contributed Session

**Contributed session (Part II)**

Dominik Edelmann, Eberhard Karls Universität Tübingen

**Finite element analysis for a diffusion equation on a harmonically evolving domain**

We present convergence results for the evolving finite element semi-discretization of a parabolic partial differential equation on an evolving bulk domain. The boundary of the domain evolves with a given velocity, which is then extended to the bulk by solving a Poisson equation. The numerical solution to the parabolic equation depends on the numerical evolution of the bulk, which yields the time-dependent mesh for the finite element method. We provide error estimates that are of optimal order in  $H^1$ -norm and present a sketch of the proof, addressing the main difficulties. Various numerical experiments are presented. In conclusion, we give an outlook on future work.

Kota Kumazaki, Nagasaki University

**Large time growth of swelling moving interfaces on a halfline**

In cold regions, buildings that are exposed to extremely low temperatures undergo freezing and build microscopic ice lenses that lead to the mechanical damage of the material. In this talk, we consider a free boundary problem as a mathematical model describing swelling of water to understand the ice lenses formation growing inside of porous materials. Our problem is posed on a halfline with a moving boundary at one of the ends, and the moving boundary conditions encode the swelling mechanism, while a diffusion equation provides water content for the swelling to take place. In this talk, we discuss the global existence and uniqueness of a solution to our problem and give the result of the large time behavior of a solution as time goes to infinity.

Martin Weiser, Zuse Institute Berlin

**Multilevel Augmented-Lagrangian Methods for Overconstrained Contact Discretizations**

We consider overconstrained formulations for stationary multi-body contact and adapted multilevel solvers of augmented Lagrange type for the resulting QPs. We discretize non-penetration by a simple symmetric, pointwise sampling without a priori choice of master and slave sides. The drawback is, that the multiplier discretization is not of the special structure provided by dual mortar spaces. This prevents the use of efficient nonlinear Gauss-Seidel smoothers as done in monotone multigrid methods. We use an augmented Lagrangian solver combined with a primal multigrid hierarchy and overlapping nonlinear block Gauss-Seidel smoothers, exploiting high arithmetic density of local penalized problems. Effective coarse grid corrections in sliding contact are achieved by level-dependent penalties.

Xinyue Zhao, University of Notre Dame

**A Free Boundary Tumor Growth Model with a Time Delay**

In this talk, I will present a non-radially symmetric tumor growth model with a time delay in cell proliferation. The time delay represents the time taken for cells to undergo replication (approximately 24h). The model is a coupled system of an elliptic equation, a parabolic equation and an ordinary differential equation. It incorporates the cell location under the presence of time delay, with the tumor boundary as a free boundary. The inclusion of a small time delay makes the system non-local, which

produces technical difficulties for the PDE estimates. I will discuss the stability results we obtained concerning this model. Through stability analysis, the results indicate that tumor with large aggressiveness parameter would trigger instability, which is biologically reasonable.

## Contributed Session

Tue 17:05–19:05 Room 4

**Contributed session (Part III)**

Nino Khatiashvili, Iv. Javakhishvili Tbilisi State University

**On the Free Boundary Problem for 2D flows**

The 2D viscous flow with large viscosity and low Reynolds number ( $Re \ll 1$ ) is considered in the half-plane. Solutions of the Stokes system are obtained, when the pressure is harmonic function. The profiles of free boundaries for the different pressures are constructed. For 2D perfect fluid the free boundary problem is reduced to the singular integral equation with the Weierstraß kernel for unknown free boundary [1]. The equation is solved numerically. The profiles of waves with peaks are constructed.

References: [1] N. Khatiashvili, On the singular integral equation with the Weierstraß kernel. *J. Complex Variables and Elliptic Equations*, Taylor & Francis, 2008, vol. 53, N 10.

Vladislav Pukhnachev, Lavrentyev Institute of Hydrodynamics

**Viscous flows with flat free boundaries**

We consider a class of exact solutions for the Navier–Stokes equations in a plane or axially symmetric case, which describes the motion in a strip or layer with one or both flat boundaries being free, while the second boundary can be a solid wall (problems A and B, respectively). In problem B, sufficient conditions for global existence of the solution or its blow up in a finite time are obtained. There are three regimes of motion in problem A: stabilization to the quiescent state, blow up in a finite time, and intermediate regime in which the viscous layer or strip expands infinitely in an infinite time. This talk is based on joint work with Elena Zhuravleva.

Athanasios Stylianou, Universität Kassel

**Modelling pattern formation on the surface of a ferrofluid**

The talk deals with patterns appearing on the free surface of a ferromagnetic fluid placed in a vertical magnetic field, undergoing a so-called Rosensweig instability. We present an existence theory for periodic and localized patterns, for a corresponding free interface problem, and tackle with the problem of regularity.

Kyung Ha, UCLA math department

**Uniformly distributing microdroplets in microparticles**

The creation of fixed-microsized droplets is important for assays and lab-on-chip applications. Recently a new method has been developed using microsized particles that capture uniform-sized droplets. For our purposes, the fluid surface will be in the shape that minimizes the surface tension energy. We use the well-known results to solve this minimal surface problem with volume constraints. By plotting the energy for a given volume we can plot the volume-energy graph for a given particle. Once we obtain the volume-energy graph of a particle, we develop a mathematical theory based on energy minimization that explains the uniform distributions of droplets. This helps us understand the reproducibility of the technology and help determine the efficient design of the particles.

Wed 09:50–11:50

Minisymposium

Wed 09:50–11:50 Room 1

**Surface PDEs (Part I)**

Organizers: C. Venkataraman, T. Ranner

Philip Brandner, RWTH Aachen University

**Stream Function Formulation of Surface Stokes Equations**

We consider surface Stokes equations on a  $C^2$  simply connected oriented stationary hypersurface  $\Gamma \subset \mathbb{R}^3$  without boundary and derive a stream function formulation based on a surface Helmholtz decomposition of these equations. This formulation results in a fourth order scalar-valued problem. For numerical simulation purposes the fourth order problem is split in two coupled second order problems, to which a TraceFEM discretization is applied. Velocity and pressure are reconstructed from the unique stream function solution. An error analysis is available. Results of numerical experiments are presented which illustrate certain properties of this approach. We draw comparisons between the stream function method and a Taylor-Hood-based TraceFEM discretization.

Argyrios Petras, Johann Radon Institute for Computational and Applied Mathematics (RICAM)

**An RBF-FD closest point method for solving PDEs on surfaces and applications to PDEs on moving surfaces**

The closest point method is an embedding method for solving PDEs on surfaces. Using a closest point representation of the surface, a constant-along-normal extension is employed to formulate the PDE in the embedded space, which can be solved numerically using standard finite difference schemes. We present a closest point method that uses finite difference schemes derived from radial basis functions (RBF-FD). Using RBF-FD, a smaller computational domain surrounding the surface is required, resulting in a decrease in the number of sampling points on the surface. The method uses RBF centers on regular grid nodes, avoiding the ill-conditioning that may arise from point clustering on the surface. Numerical tests include a number of applications on static and moving surfaces.

Marino Arroyo, Universitat Politècnica de Catalunya-BarcelonaTech

**Free boundary problems in cell mechanobiology**

Interfacial mechanical and chemical phenomena control the shape, motion, mechanical properties, and many functions in animal cells. I will present mathematical models and simulations of membrane shaping by proteins, cytoskeletal dynamics, and adhesion dynamics pertinent to cell mechanobiology.

Minisymposium

Wed 09:50–11:50 Room 2

**Nonlocal free boundary problems (Part I)**

Organizers: E. Valdinoci, J.-F. Rodrigues

Alexander Nazarov, St. Petersburg Dept of Steklov Institute and St. Petersburg State University

**Variational Inequalities for the Fractional Laplacians**

We study the obstacle problems for the Dirichlet and Navier fractional Laplacians of order  $s \in (0, 1)$  in a bounded Lipschitz domain  $\Omega \subset \mathbb{R}^n$ , under mild assumptions on the data. We obtain some regularity results and compare the solutions for two types of Laplacians. This talk is mainly based on the papers R. Musina, A.I. Nazarov, K. Sreenadh, *Potential Analysis*, V. 46 (2017), N3, and R. Musina, A.I. Nazarov, *Comp. Math. and Math. Phys.*, V. 57 (2017), N3. The author was partially supported by RFBR grant 20-01-00630a.

Aram Karakhanyan, University of Edinburgh

**A nonlocal free boundary problem with Wasserstein distance**

We study the probability measures  $\rho \in \mathcal{M}^2$  minimizing the functional

$$J[\rho] = \iint \log \frac{1}{|x-y|} d\rho(x)d\rho(y) + d^2(\rho, \rho_0),$$

where  $\rho_0$  is a given probability measure and  $d(\rho, \rho_0)$  is the 2-Wasserstein distance of  $\rho$  and  $\rho_0$ .  $J[\rho]$  appears in aggregation models when the movement of particles is advanced by the potential  $-\log|x| * \rho$ . We prove the existence of minimizers  $\rho$  and show that the potential  $U^\rho = -\log|x| * \rho$  solves a degenerate *obstacle problem*, the obstacle being the transport potential. Among other things, we study the regularity of minimizers and the structure of singular set of free boundary.

Xavier Fernández-Real, EPFL

**Stable cones in the thin one-phase problem**

The fractional or thin one-phase problem is the study of minimizers of the functional

$$\mathcal{J}_s(u) = [u]_{H^s(B_1)}^2 + |\{u > 0\} \cap B_1|.$$

The aim of this talk is to study homogeneous stable solutions to the fractional one-phase free boundary problem. The problem of classifying stable (or minimal) homogeneous solutions in dimensions  $n \geq 3$  is completely open. In this context, axially symmetric solutions are expected to play the same role as Simons' cone in the classical theory of minimal surfaces, but even in this simpler case the problem is open. The goal of this talk is twofold and is based on a recent work with X. Ros-Oton. On the one hand, we present our first main contribution: the stability condition for the thin one-phase problem. Quite surprisingly, this requires the use of "large solutions" for the fractional Laplacian, which blow up on the free boundary. On the other hand, using our new stability condition, we show that any axially symmetric homogeneous stable solution in dimensions  $n \leq 5$  is one-dimensional, independently of the parameter  $s \in (0, 1)$ .

Juan Luis Vázquez, Universidad Autónoma de Madrid

**Nonlocal free boundary problems of porous medium type**

The property of finite propagation and existence of free or free boundaries is well known in the theory of nonlinear elliptic and parabolic equations of nonlinear degenerate diffusive type. When the Laplacian type diffusivity is replaced by nonlocal operators, like fractional operators, such a property is or is

not conserved depending on the balance between the degenerate nonlinearities and the nonlocal character of the fractional diffusion.

Minisymposium

Wed 09:50–11:50 Room 3

### Stochastic free boundary problems (Part I)

Organizers: A. Djurdjevac, B. Gess, G. Grün

Lubomir Banas, Bielefeld University

#### Convergent numerical approximation of the stochastic porous media equation in higher dimensions.

We propose a fully discrete numerical scheme for the approximation of the stochastic porous media equation (SPME) in its very weak formulation. Using the monotonicity properties of the very weak formulation we show that the proposed numerical scheme converges to the strong solution of SPME. We construct a non-standard finite element based spatial approximation of the problem which yields an efficient numerical algorithm in the spatial dimension  $d = 2, 3$ . We present numerical experiments to illustrate the behavior of the proposed numerical algorithm. The talk is based on a joint work with Benjamin Gess and Christian Vieth.

Nicolas Dirr, Cardiff University

#### Probability and Analysis in Multi-Scale Systems

We give an overview over recent results, both rigorous and numerical, for multiscale problems where there is nonlinear diffusion on the macro-scale and a high-dimensional stochastic system on the micro-scale.

Hubertus Grillmeier, Friedrich-Alexander-Universität Erlangen-Nürnberg

#### Stochastic porous-medium equations: a sufficient condition for a.s. instantaneous propagation of free boundaries

For stochastic porous-medium equations with multiplicative linear source-term noise, Fischer and Grün recently formulated sufficient conditions on the growth of initial data close to the free boundary which guarantee the occurrence of a waiting time phenomenon, i.e., a delayed onset of propagation of the free boundary. In this talk, we study the reverse problem. We rigorously identify new growth conditions sufficient for instantaneous spreading of solutions almost surely. For this, we derive an upper bound for the average time at which solutions begin to spread. Technically, we combine Itô-formulas including random stopping times with weighted integral estimates. It is worth mentioning that these results are not confined to Lipschitz-regular noise terms.

Sebastian Hensel, Institute of Science and Technology Austria (IST)

#### Finite time extinction for the 1D stochastic porous medium equation with transport noise

We establish finite time extinction with probability one for the Cauchy-Dirichlet problem of the 1D stochastic porous medium equation with Stratonovich transport noise and compactly supported smooth initial datum. This is expected since Brownian motion has average spread rate  $O(t^{\frac{1}{2}})$  whereas the support of solutions to the deterministic PME grows only with rate  $O(t^{\frac{1}{m+1}})$ . I will discuss the three main ingredients for a rigorous implementation of this heuristic: i) a contraction principle up to time-dependent shift for Wong-Zakai type approximations, ii) the transformation to a deterministic PME with two copies of a Brownian path as the lateral boundary, and iii) the construction of a maximal subsolution for the latter problem in terms of the pressure variable.

## Minisymposium

Wed 09:50–11:50 Room 4

**Geometric problems (Part II)**

Organizers: M. Novaga, Y. Giga

Paola Pozzi, University of Duisburg-Essen

**On the elastic flow networks**

We call a network the union of sufficiently smooth curves that meet at junctions. The simplest configuration is a three-network: this is the union of three curves that meet at one or two junctions. In the first case we speak of a triod, in the second of a theta-network. Networks and flow of networks arise naturally in the study of multiphase systems and when considering the dynamic of their interfaces. In this talk I will present recent results for the elastic flow of three-networks.

Lucia Scardia, Heriot-Watt University

**Equilibrium measures for nonlocal energies: The effect of anisotropy**

Nonlocal energies are continuum models for large systems of particles with long-range interactions. Under the assumption that the interaction potential is radially symmetric, several authors have investigated qualitative properties of energy minimisers. But what can be said in the case of anisotropic potentials? Starting from the example of dislocations in materials science, I will discuss some results and partial answers on how anisotropy affects the shape and dimension of the support of minimisers. This is based on work in collaboration with Jose Antonio Carrillo, Joan Mateu, Maria Giovanna Mora, Luca Rondi and Joan Verdera.

Ulisse Stefanelli, University of Vienna

**Rate-independent evolution of sets**

I will present a model for the rate-independent evolution of sets with finite perimeter. The evolution is driven by a given time-dependent forcing set, acting as a one-sided constraint. Solutions result from the quasistatic competition between perimeter minimization and minimization of volume changes. This idealized setting arises in connection with delamination models in the so-called brittle limit and provides a sort of rate-independent mean-curvature flow. I will comment on this point and present existence and approximation results. In the two-dimensional case, the geometry and regularity of solutions will be assessed and some numerical results will be presented. This is a joint work with Riccarda Rossi (Brescia) and Marita Thomas (Berlin).

Mathias Wilke, Martin-Luther University Halle-Wittenberg

**On the Navier–Stokes equations on surfaces**

We consider a model for the motion of an incompressible viscous fluid that completely covers a smooth, compact hypersurface  $\Sigma$  in  $\mathbb{R}^{d+1}$  and flows along  $\Sigma$ . Several questions related to well-posedness and the qualitative behavior are addressed. This is a joint work with Jan Prüss and Gieri Simonett (Vanderbilt University, Nashville, TN, USA)

Wed 12:45–14:45

Minisymposium

Wed 12:45–14:45 Room 1

**Regularity of free boundaries (Part I)**

Organizers: H. Shahgholian, M. Smit Vega Garcia

Jinwan Park, Seoul National University

**The Regularity Theory for the Double Obstacle Problems**

In this talk, I will introduce the regularity of the free boundary for the double obstacle problems. Precisely, I will discuss the interior  $C^1$  regularity of the free boundary for the elliptic and parabolic double obstacle for linear and fully nonlinear operator. Furthermore, I will introduce the  $C^1$  regularity of the free boundary near fixed boundary for Laplacian problem. The contents are based on the paper and pre-prints, joint works with Professor Henrik Shahgholian and Ki-Ahm Lee.

Sunghan Kim, KTH Royal Institute of Technology

**On an elliptic free boundary problem arising from jump of conductivity**

When a composite material changes its conductivity along the phase transition, it can be mathematically modelled by an elliptic PDE whose coefficient changes discontinuously along certain level surface of the associated solution. Such a level surface becomes the free boundary. In this talk, we shall consider one such problem given by

$$\operatorname{div}(A(x, u)\nabla u) = 0,$$

where conductivity matrix  $A(x, u)$  has a jump across the zero level set,  $\{u = 0\}$ , of a given solution  $u$ , but is smooth elsewhere. We shall discuss the regularity of the free boundary near a non-degenerate point, the structure of the singular set, and the behaviour of the free boundary near a fixed boundary. This is partially based on a joint work with Ki-Ahm Lee and Henrik Shahgholian.

Nicola Soave, Politecnico di Milano

**Free boundary problems in the spatial segregation of competing systems**

In this talk we present some results concerning the spatial segregation in systems with strong competition. In particular, we focus on two different (but strongly related) issues: long-range segregation models and systems characterized by asymmetric diffusion. The content of the talk is part of ongoing project with H. Tavares, S. Terracini and A. Zilio.

Simon Eberle, University of Duisburg-Essen

**On global solutions of the obstacle problem**

In this talk we investigate global solutions of the classical obstacle problem. I give a partial result towards a conjecture by H. Shahgholian ('92) saying that the coincidence sets of global solutions of the obstacle problem are in the closure of ellipsoids, i.e. ellipsoids, paraboloids, cylinders with one of the two as basis, or half-spaces. I give a short proof of the known result that bounded coincidence sets of global solutions of the obstacle problem that have non-empty interior are ellipsoids. My main and new result is that in dimensions greater or equal to 6 coincidence sets of global solutions, that are not constant in any direction are paraboloids if they have non-empty interior.

Minisymposium

Wed 12:45–14:45 Room 2

**Numerical methods for surface-bulk problems (Part II)**

Organizers: A. Reusken, E. Bänsch

Stefan Metzger, FAU Erlangen-Nürnberg

**On the numerical treatment of Cahn–Hilliard equations with dynamic boundary conditions**

This talk addresses the numerical treatment of a family of models coupling Cahn–Hilliard type equations in the domain and on the boundary. All of these models describe an evolution minimizing the sum of Ginzburg–Landau type free energies in the bulk and on the surface, but differ in the equilibration rate of the corresponding chemical potentials. The system describing the limit case of instantaneous equilibration was proposed and analysed by Goldstein, Miranville, and Schimperna (Physica D, 2011), while the case of no equilibration was investigated by Liu and Wu (Arch. Ration. Mech. Anal, 2019). We will present a stable, convergent, fully discrete finite element scheme that is able to deal with the complete class of models. The results presented in this talk are based on a joint work with Patrik Knopf (Universität Regensburg), Kei Fong Lam (Hong Kong Baptist University), and Chun Liu (Illinois Institute of Technology).

Robert Nürnberg, University of Trento

**Fluidic two-phase biomembranes: Numerical analysis and computations**

A parametric finite element approximation of a fluidic membrane, whose evolution is governed by a surface Navier–Stokes equation coupled to bulk Navier–Stokes equations, is presented. The elastic properties of the membrane are modelled with the help of curvature energies of Willmore and Helfrich type. We allow for two different phases to be present on the membrane, with their own elastic properties, giving rise to an additional line energy. This is modelled with the help of a phase field that evolves according to a surface Cahn–Hilliard equation. In this talk we investigate some desirable properties of the scheme, including stability and good mesh properties, and present several numerical simulations.

Thomas Ranner, University of Leeds

**Numerical analysis of a coupled bulk-surface problem in an evolving domain**

Based on an abstract theory for continuous in time finite element discretisations of partial differential equations posed in evolving domains, I will present numerical analysis of a simple coupled bulk-surface problem. The problem couples a linear parabolic equation in the evolving bulk domain to a linear parabolic equation posed on the boundary surface of the domain. Evolving bulk and surface isoparametric finite element spaces defined on evolving triangulations will be developed. Optimal a priori bounds are shown under usual assumptions on the geometry and solution of the partial differential equation. We conclude with a numerical example.

Alfred Schmidt, University of Bremen

**Phase transitions with free surface flow: a 3D FEM with a Young–Laplace formulation**

Problems with melting and solidification including free surface flow of the melt arise in models for applications like welding or laser melted material accumulation. They pose special challenges to the numerical discretization and solution. Both sharp interface and implicit phase boundary formulations with free surface flow have pros and cons in some situations, a combination of both may lead to stable method, but is applicable only



in special cases. Especially in 3D, this is often not the case. We propose a model with a Young-Laplace formulation for the capillary melt surface shape, and present a corresponding 3D finite element method and numerical results. This is joint work with Henrik Ücdemir, Eberhard Bänsch, Mischa Jahn, and Andreas Luttmann

Minisymposium

Wed 12:45–14:45 Room 3

### Optimization and control of FBPs (Part II)

Organizers: M. Hintermüller, M. Hinze

René Pinnau, TU Kaiserslautern

#### Optimal Control of FBVPs in Filter Production [canceled]

In this talk we present different FBVPs in filter production and corresponding optimal control problems. These include the production of the fibre material as well as the control of the filtration process. The problems are analysed and we use the adjoint variables to derive the derivative information which is needed for the respective numerical solution. Further, we discuss possible relaxations and approximations.

Dmytro Strelnikov, TU Ilmenau

#### Optimal control for crack-free pulsed laser beam welding of aluminium alloys

The laser welding process consists of the melting and the solidification stages. Our goal is to avoid the material cracking due to its fast solidification by choosing the laser power mode. The process may be modeled as a BVP with the heat equation at its core. The equation coefficients are temperature dependent in order to properly take different modes of heat transfer and the phase transition into account. The interface between the solid and the liquid regions is not known at the outset of the problem and is time dependent. The mathematical formulation of the problem is set up and the numerical model is implemented using the finite-element software FEniCS. We solve an optimal control problem for the laser power mode by minimizing the cost functional based on the solidification velocity.

Patrik Knopf, Universität Regensburg

#### Phase-field methods for spectral shape and topology optimization

We optimize a selection of eigenvalues of the Laplace operator with Dirichlet or Neumann boundary conditions by adjusting the shape of the domain on which the eigenvalue problem is considered. Here, a phase-field function is used to represent the shapes over which we minimize. The idea behind this method is to modify the Laplace operator by introducing phase-field dependent coefficients in order to extend the eigenvalue problem on a fixed design domain containing all admissible shapes. The resulting shape and topology optimization problem can then be formulated as an optimal control problem with PDE constraints in which the phase-field function acts as the control. For this optimal control problem, we present first-order necessary optimality conditions and several numerical simulations.

Björn Baran, Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg

#### Feedback Control of the Stefan Problem

We discuss the feedback stabilization of a two-dimensional two-phase Stefan problem coupled with (Navier-)Stokes equations. The Stefan problem can model solidification and melting of pure materials and gets its name from the purely algebraic Stefan condition which describes the coupling between the temperature of the material and its melting process. After linearization and discretization, the stabilization problem results in a non-autonomous differential Riccati equation (DRE) with differential-algebraic structure. We present novel techniques to treat the specific structure of this coupled problem in a DRE solver and illustrate its performance using numerical examples.

Wed 12:45–14:45 Room 4

**Interfaces in fluids (Part III)**

Organizers: H. Abels, H. Garcke, M. Wilke

Andrea Giorgini, Indiana University

**Well-posedness of the Abels-Garcke-Grün model for two-phase flows**

The Abels-Garcke-Grün (AGG) system is a diffuse interface model describing multi-phase fluid flows. This was proposed as a generalization of the well-known Model H in the case of unmatched homogeneous densities of the fluids. The model consists of a Navier–Stokes–Cahn–Hilliard system characterized by a concentration-dependent density and an additional flux term due to interface diffusion. Using the method of matched asymptotic expansions, the sharp interface limit of the AGG model corresponds to the two-phase Navier–Stokes equations. In the literature, the analysis of the AGG system has been mainly focused on the existence of weak solutions. In this talk, I will present some recent results on the existence, uniqueness and stability of strong solutions for the AGG model.

Ian Tice, Carnegie Mellon University

**Traveling wave solutions to the free boundary Navier–Stokes equations**

Consider a layer of viscous incompressible fluid bounded below by a flat rigid boundary and above by a moving boundary. The fluid is subject to gravity, surface tension, and an external stress that is stationary when viewed in coordinate system moving at a constant velocity parallel to the lower boundary. The latter can model, for instance, a tube blowing air on the fluid while translating across the surface. In this talk we will detail the construction of traveling wave solutions to this problem, which are themselves stationary in the same translating coordinate system. While such traveling wave solutions to the Euler equations are well-known, to the best of our knowledge this is the first construction of such solutions with viscosity. This is joint work with Giovanni Leoni.

Maurizio Grasselli, Politecnico di Milano

**Nonlocal Cahn–Hilliard–Hele–Shaw systems**

I intend to present some recent results on nonlocal Cahn–Hilliard–Hele–Shaw systems focusing on a model characterized by nonconstant mobility, singular potential, and nonconstant kinematic viscosity. Well-posedness and regularity issues will be discussed. These results have been obtained jointly with C. Cavaterra and S. Frigeri (Università degli Studi di Milano, Italy).

Wed 16:15–18:15

Minisymposium

Wed 16:15–18:15 Room 1

**Numerical methods for geometric PDEs (Part II)**

Organizers: R. Nochetto, R. Nürnberg

Antonin Chambolle, CNRS and CEREMADE, Université Paris-Dauphine, PSL, France

**Learning better discretizations of the total variation**

In this talk I will describe two approaches to improve consistent discretizations of the total variation, (1) by designing a periodic pattern and homogenization (2) by learning a template for enforcing constraints in the dual formulation. This is joint work with T. Pock (Graz), L. Kreutz (Münster)

Klaus Deckelnick, Otto-von-Guericke-Universität Magdeburg

**A novel finite element scheme for anisotropic curve shortening flow**

We consider the motion of parametric curves by anisotropic curve shortening flow. We propose a strictly parabolic system of PDEs for the position vector whose solution satisfies the corresponding evolution law. This system is discretized by linear finite elements and optimal  $H^1$  error bounds are proved in the semidiscrete case. We present numerical experiments which show that the scheme has good properties with respect to the distribution of mesh points. This is joint work with Robert Nuernberg.

Hanne Hardering, TU Dresden

**Tangential errors of tensor surface finite elements**

We discretize a tangential tensor field equation using a surface-finite element approach with a penalization term to ensure almost tangentiality. It is natural to measure the quality of such a discretization intrinsically, i.e. to examine the tangential convergence behavior in contrast to the normal behavior. We show optimal order convergence with respect to the tangential quantities in particular for an isogeometric penalization term that is based only on the geometric information of the discrete surface. This is joint work with S. Praetorius.

Martin Rumpf, Universität Bonn

**Finite Element Approximation of Large Scale Isometric Deformations of Parametrized Surfaces**

In this talk the numerical approximation of isometric deformations of thin elastic shells is discussed. To this end, for a thin shell represented by a parametrized surface it is shown how to transform the stored elastic energy for an isometric deformation such that - similar to the model for plate deformations - the highest order term is quadratic. For this reformulation existence of optimal isometric deformations is shown. A finite element approximation is obtained using the Discrete Kirchhoff Triangle (DKT) approach and the convergence of discrete minimizers to a continuous minimizer is demonstrated. In that respect, this paper generalizes the results by Sören Bartels for the approximation of bending isometries of plates. A Newton scheme is applied to experimentally verified the convergence properties and discuss characteristics of isometric deformations. The talk is based on joint work with Stefan Simon and Christoph Smoch from Bonn.

Minisymposium

Wed 16:15–18:15 Room 2

**Free boundary models in active matter**

Organizer: L. Berlyand

Leonid Berlyand, Penn State University

**Asymptotic Stability in an FBP model of Contraction-Driven Cell Motion**

We introduce a free boundary model of the onset of motion of a living cell (e.g. a keratocyte) driven by myosin contraction with focus on a transition from unstable radial stationary states to stable asymmetric moving states. This model generalizes a previous 1D model (Truskinovsky et al.) by combining a Keller-Segel model, a Hele-Shaw boundary condition and the Young-Laplace law with a nonlocal regularizing term which precludes blow up or collapse by ensuring that membrane-cortex interaction is sufficiently strong. We show that this model has a family of asymmetric traveling wave solutions bifurcating from a family of stationary states. Our goal is to establish observable steady cell motion with constant velocity. Mathematically, this amounts to proving stability of the traveling wave solutions, which requires generalization of the standard notion of stability. Our main result is establishing nonlinear asymptotic stability of traveling solutions. To this end, we derive an explicit asymptotic formula for the stability-determining eigenvalue from asymptotic expansions in small speed. This formula greatly simplifies the numerical computation of the sign of this eigenvalue and reveals the physics underlying onset of the cell motion and stability of moving states. This is joint work with V. Rybalko and C. A. Safsten.

Ricard Alert, Princeton University

**Stability of chemotactic fronts**

In contexts ranging from embryonic development to bacterial ecology, cell populations migrate chemotactically along self-generated chemical gradients, often forming a propagating front. The motion of such chemotactic fronts constitutes a free-boundary problem where the interface is driven by a coupling between the cell density field and the gradient of the chemical concentration field. I will analytically show that the linear stability of chemotactic fronts to morphological perturbations is determined by limitations in the ability of individual cells to sense and respond to the chemical gradient. Specifically, I will argue that cells at a bulging part of the front are exposed to a smaller gradient, which slows them down and promotes stability, but they respond more strongly to the gradient, which speeds them up and promotes instability. We predict that this competition leads to chemotactic fingering when sensing is limited at low chemical concentrations, and to stable flat fronts when it is instead limited at high concentrations. Guided by this finding and by experimental data on bacterial chemotaxis, I will suggest that the cells' sensory machinery might have evolved to ensure stable front propagation. Finally, as sensing of any stimuli is necessarily limited, the principle of sensing-induced stability may operate in other types of directed migration such as durotaxis, electrotaxis, and phototaxis. This is joint work with Sujit S. Datta at Princeton University.

Jaume Casademunt, University of Barcelona

**Symmetry breaking and motility of cells and tissues**

Understanding the physical mechanisms that govern cell motility and collective cell migration is of great importance to a variety of challenging biological problems, from embryogenesis to tissue repair and cancer invasion. To this aim, both individual cells and cell colonies are often modeled as quasi-two-dimensional droplets of active fluids that define free-boundary problems. Here we provide a brief overview of several such

models, with particular focus on the interplay between the mechanisms of symmetry breaking and directed motion. We discuss prototype models for individual cells where motility originates on a spontaneously broken symmetry, either from a purely morphological instability of the boundary, or involving an inhomogeneous distribution of a chemical species that controls the activity. We then discuss a recently introduced free-boundary problem based on a hydrodynamic model for cohesive cell tissues that spread and migrate on a substrate. The model has been tested successfully in experiments of epithelial spreading and exhibits a morphological instability. It has been shown to explain collective cell migration with an externally broken symmetry (collective durotaxis). However, it remains an interesting open question whether such a free-boundary problem for tissues gives rise to spontaneous motion of cell colonies solely driven by a morphological instability.

**Carles Blanch-Mercader**, Institut Curie  
**Morphogenesis of anisotropic cell monolayers: quantifying material properties by analyzing integer topological defects**

Tissues made of anisotropic cells can exhibit features of liquid crystals, such as long-ranged orientational order and topological defects. During the development of organisms, such features often influence shape formation. However, the linkage between geometry, topological defects and tissue mechanics remains largely unexplored. First, I will show how we build on the physics of liquid crystals to determine material parameters of cell monolayers. In particular, we use a hydrodynamical description of a 2d active liquid crystal to study the steady-state mechanical patterns around integer topological defects. This model predicts a transition from non-rotating asters to rotating spirals. We apply our approach to spindle-shaped myoblast cell monolayers in small circular confinements, which spontaneously form isolated aster or spiral topological defects. Finally, I will discuss how these findings can help one interpreting a secondary transition from 2d cell monolayers to 3d multicellular structures, such as mounds or protrusions, and I will present some open challenges in free-boundary models whereby couplings between geometry and orientational order dominate. This is joint work with P. Guillamat, A. Roux, K. Kruse. P. Guillamat and the presenter contributed equally to this work.

Wed 16:15–18:15 Room 3

**Regularity of free boundaries (Part II)**  
 Organizers: H. Shahgholian, M. Smit Vega Garcia

**Mark Allen**, Brigham Young University  
**Free Boundary Problem on a Cone**

We consider a free boundary problem on cones depending on a parameter  $c$  and study when the free boundary is allowed to pass through the vertex of the cone. When the cone is two-dimensional, the free boundary always avoids the vertex of the cone. However, when the cone is three-dimensional, the free boundary may pass through the vertex of the cone if  $c$  is small. For large values of  $c$ , the free boundary avoids the vertex. Our result is analogous to a result of Morgan that classifies when an area-minimizing surface on a cone passes through the vertex. This is joint work with Blake Barker, Jason Gardiner, and Mingyan Zhao.

**Dennis Kriventsov**, Rutgers University  
**Optimal regularity for an obstacle problem with log singularity**

I will present an optimal regularity result for solutions to the semilinear equation

$$\Delta u = (-\log u^+)1_{\{u>0\}} - (-\log u^-)1_{\{u<0\}}.$$

In particular, solutions have log-Lipschitz derivatives. This problem has similar structure to the classical two-phase obstacle problem (and has the same blow-up limits), but the right-hand side's unfavorable monotonicity in  $u$  obstructs most arguments from carrying over. The method I describe is based on a careful analysis of Weiss-type energies in this setting. This is based on joint work with Henrik Shahgholian.

**Edgard Pimentel**, PUC-Rio  
**Nodal sets for elliptic equations in the double divergence form**

We examine nodal sets for elliptic equations in the double divergence form. We prove that the nondegenerate part of the nodal set inherits the regularity of the solutions. Then, we focus on the structure of its singular part. Finally, we impose further conditions on the coefficients and show that nodal sets do not disconnect the domains. We end the talk with comments on further directions of research.

**Murat Akman**, University of Essex  
**An eigenvalue problem for  $p$ -Laplace equation with an application to Minkowski type problem**

We study existence and uniqueness of homogeneous solutions to  $p$ -Laplace equation,  $p$  fixed,  $n - 1 < p < \infty$ ,  $n \geq 2$ , when  $u$  is a solution in  $K(\alpha) \subset \mathbb{R}^n$  where

$$K(\alpha) := \{x = (x_1, \dots, x_n) : x_1 > \cos \alpha |x|\}$$

for fixed  $\alpha \in (0, \pi]$ , with continuous boundary value zero on  $\partial K(\alpha) \setminus \{0\}$ . In our main result we show that if  $u$  has continuous boundary value 0 on  $\partial K(\pi)$  then  $u$  is homogeneous of degree  $1 - (n - 1)/p$  when  $p > n - 1$ . We then apply this result to study the regularity of the Minkowski type problem.

Thu 09:50–11:50

## Minisymposium

Thu 09:50–11:50 Room 1

**Asymptotic approaches to interface dynamics (Part I)**

Organizers: J. José López Velázquez, J. King

John Ward, Loughborough University

**Moving interfaces in a model of interacting mobile and immobile bacterial populations**

Bacteria typically exist in two phases, a mobile state (e.g. swimming) or in a sessile state (forming colonies on solid-fluid interfaces known as biofilms). The interaction and switching between the mobile and immobile states can greatly influence the rate of biofilm expansion, particularly in the early stages of development. In this talk we discuss a simple model describing biofilms as a growing viscous fluid on a surface in a static fluid media consisting of swimming bacteria. Applying a thin-film reduction, a coupled, two-phase system of PDEs is derived consisting of Fisher's type equations for the mobile cells and for the biofilm; the latter consisting of a fourth-order diffusion like term. The system generates a moving boundary problem for the biofilm contact line. The key results will be presented, in particular how the growth and diffusive properties of each of the phases combine to affect the speed of the moving boundary.

Amrita Ghosh, Institute for Applied Mathematics, University of Bonn

**On Shikhmurzaev's approach to the Contact Line Problem**

I will discuss the general moving contact line problem, arising from many fluid mechanics phenomena. The classical formulation of this model with the no-slip (Dirichlet) boundary condition for velocities at the liquid-solid interface gives rise to a non-integrable singularity of the shear stress, which is known as the "moving contact line paradox". Y. Shikhmurzaev proposed (1993) a different approach to solve this issue. This approach, apart from the classical conservation equations, formulates the boundary conditions for the equations in the bulk phases in an elaborate way, which can be viewed as a generalization of the existing models in the literature, although generated from a different underlying theory. After giving an overview of this model, I would like to derive a thin film approximation of Shikhmurzaev's model to compare with the other solution model of the contact line problem.

Diego Alonso Oran, Universität Bonn

**Asymptotic shallow models arising in magnetohydrodynamics**

In this talk, we derive new shallow asymptotic models for the free boundary plasma-vacuum problem governed by the magnetohydrodynamic equations which are vital when describing large-scale processes in flows of astrophysical plasma. More precisely, we present the magnetic analogue of the 2D Green-Naghdi equations for water waves in the presence of weakly sheared vorticity and magnetic currents. Our method is inspired by ideas for hydrodynamic flows to reduce the two/three-dimensional dynamics of the problem to a finite cascade of equations which can be closed at the precision of the model.

Scott McCue, Queensland University of Technology

**Interfacial dynamics for axially symmetric Darcy flow and related conduction limited melting problems**

I will discuss a moving boundary model for the evolution of an axially symmetric bubble of inviscid fluid in a homogeneous porous medium otherwise saturated with a viscous fluid. The model is a higher dimensional analog of Hele-Shaw flow, but also applies for a one-phase quasi-steady Stefan problem for

melting a crystal dendrite. One major focus is the development of pinch-off singularities characterized by a blow-up of the interface curvature and the bubble subsequently breaking up into two. We show how similarity solutions can describe pinch-off, whereby the minimum radius behaves as a power law in time with exponent  $1/3$  just before and after pinch-off has occurred. For a geometry involving a cylindrical tube, there appears to be time-dependent solutions that are like axially symmetric Saffman-Taylor fingers or Taylor-Saffman bubbles. Further, if time permits, I will also summarise some analysis for the related problem of melting a crystal dendrite, including asymptotic results in the limit that the dendrite melts completely.

## Minisymposium

Thu 09:50–11:50 Room 2

**Stochastic free boundary problems (Part II)**

Organizers: A. Djurdjevac, B. Gess, G. Grün

Peter Nejjar, Universität Bonn

**Fluctuations of a microscopic shock wave in the Burgers equation**

One of the most basic PDEs is the one-dimensional Burgers equation. As is well known, this equation can be solved by the method of characteristics. The asymmetric simple exclusion process (ASEP) is an interacting particle system whose large time particle density solves the Burgers equation. Furthermore, ASEP provides a stochastic analogue to the characteristics: This analogue is the second class particle. Second class particles have proven invaluable to understand fluctuations in random growth models belonging to the Kardar-Parisi-Zhang (KPZ) universality class such as the KPZ equation. Here we consider ASEP with a shock in the Burgers equation, and obtain the large time fluctuations of the second class particle following the shock.

Benjamin Seeger, Université Paris - Dauphine

**Scaling limits and homogenization of Hamilton-Jacobi equations with stochastic forcing**

I will present results concerning the homogenization of Hamilton-Jacobi equations forced by stochastic and/or mixing noise. Emphasis will be placed on the wide variety of possible limiting problems that arise, depending on the scaling regime and the structure of the equations. In particular, the effective equation can be stochastic or deterministic, with the noise having an enhancement effect in the latter case, or explosion can occur. I also discuss some new regularity results, which are an important tool in some of the proofs and are of independent interest.

Ana Djurdjevac, Zuse Institute Berlin

**PDEs and random fields on hypersurfaces with random velocity**

The uncertainty in the mathematical modeling appears for various reasons, we are especially interested when it comes from the geometric aspect, i.e. curved domain. We study the PDEs posed on curved domains that evolve with a given random velocity. Utilizing the domain mapping method, we transfer the problem into a PDE with random coefficients on a fixed domain. For numerical analysis, we consider surface FEM coupled with MC sampling. A natural question in this setting is how to represent a random field on a hypersurface. We present the ongoing work in this direction that is based on the wavelet type expansion. This is a joint work with L. Church, C. Elliott, M. Bachmayr.

Thu 12:45–14:45

Minisymposium

Thu 12:45–14:45 Room 1

**Free boundary problems for nonlinear hyperbolic/mixed-type PDEs (Part II)**

Organizer: G.-Q. Chen

Mikhail Feldman, UW-Madison

**Uniqueness and stability for shock reflection problem**

We discuss shock reflection problem for compressible gas dynamics, von Neumann conjectures on transition between regular and Mach reflections, and existence of regular reflection solutions for potential flow equation. Then we will talk about recent results on uniqueness and stability of regular reflection solutions for potential flow equation in a natural class of self-similar solutions. The approach is to reduce the shock reflection problem to a free boundary problem for a nonlinear elliptic equation, and prove uniqueness by a version of method of continuity. A property of solutions important for the proof of uniqueness is convexity of the free boundary. This talk is based on joint works with G.-Q. Chen and W. Xiang.

Paolo Secchi, University of Brescia

**Geometric optics for surface waves on the plasma-vacuum interface**

In this talk we consider the free boundary problem in three space dimensions for a plasma-vacuum interface in ideal incompressible magnetohydrodynamics. Unlike the classical statement, where the vacuum magnetic field obeys the div-curl system of pre-Maxwell dynamics, we do not neglect the displacement current in the vacuum region and consider the Maxwell equations for electric and magnetic fields. Our aim is to construct weakly nonlinear, highly oscillating solutions to this plasma-vacuum interface problem. Under a necessary and sufficient stability condition for a piecewise constant background state, we construct by a geometric optics approach approximate solutions, at any arbitrarily large order of accuracy, when the initial discontinuity displays high frequency oscillations. As evidenced in earlier works, high frequency oscillations of the plasma-vacuum interface solution give rise to surface waves on either side of the interface. Such waves decay exponentially in the normal direction to the interface and, in the weakly nonlinear regime that we consider here, their leading amplitude is governed by a nonlocal Hamilton-Jacobi type equation, as for Rayleigh waves in elastodynamics and current-vortex sheets in MHD. This is a joint work with Yuan Yuan (CAMIS, South China Normal Univ.)

Dehua Wang, University of Pittsburgh

**Inviscid limit of compressible viscoelastic equations with the no-slip boundary condition**

The inviscid limit for the two-dimensional compressible viscoelastic equations on the half plane is considered under the no-slip boundary condition. It is well-known that for the corresponding inviscid limit of the compressible Navier–Stokes equations with the no-slip boundary condition, one does not expect the uniform energy estimates of solutions due to the appearance of strong boundary layers. Our results show that the deformation tensor plays an important role in the vanishing viscosity process and can prevent the formation of strong boundary layers. This is a joint work with Feng Xie.

Steve Shkoller, University of California, Davis

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Minisymposium

Thu 12:45–14:45 Room 2

**Free boundary problems in cell biology (Part I)**

Organizers: C. Gräser, M. Röger

Helmut Abels, Universität Regensburg

**A Bulk-Surface Model for Lipid Raft Formation and its Sharp Interface Limit**

We discuss a model for phase separation on biological membranes for the evolution of lipid rafts which was recently proposed by Garcke, Rätz, Röger and Kampmann. The model is an extended Cahn–Hilliard equation which contains additional terms to account for the active transport processes. We discuss its sharp interface limit and existence of weak solutions for the limit model.

Martin Burger, FAU Erlangen-Nürnberg

**A Double Interface Model for Cell Blebbing**

In this talk we will discuss the mathematical modelling of cell blebbing phenomena, which is characterized by the interaction of two interfaces, the cell membrane and the actin cortex. This is supplemented by a fluid structure interaction with the intracellular flow and activation and deactivation of linker proteins. We discuss sharp and diffuse interface models and their properties.

Luca Lussardi, Politecnico di Torino

**Membrane shapes that minimize the multiphase Canham-Helfrich energy**

The Canham-Helfrich energy is widely used to describe the elastic properties of biological membranes at the sub-cellular level. The membranes are modeled as surfaces in the 3-dimensional space and their shape minimize the energy which penalizes the curvatures of the surface. This energy can be generalized to the multiphase case in order to model also heterogeneous biological membranes. First of all I will review the rotational symmetric case (Choksi-Morandotti-Veneroni, 2013). If no symmetry of the minimizers is assumed, the problem requires other tools. I will discuss existence of single and multiphase minimizers under area and enclosed volume constraints. This is a joint work with K. Brazda and U. Stefanelli both from University of Vienna.

Anna Logiotti, University of Bonn

**A free boundary problem modeling cell polarization**

We study a model for cell polarization as a response to an external signal which results in a system of PDEs for different variants of a protein on the cell surface and interior respectively. We study the evolution of this model for several reaction rates on the membrane and the diffusion coefficient inside the cell being large. Under suitable scaling limits, solutions converge to solutions of obstacle type problems, for which we show uniqueness. Further, we prove the global stability of steady states. For the steady states the occurrence of polarization has been shown by Niethammer, Röger and Velázquez (2019) for sufficiently small total mass of protein.

## Minisymposium

Thu 12:45–14:45 Room 3

**Nonlocal free boundary problems (Part II)**

Organizers: E. Valdinoci, J.-F. Rodrigues

Lisa Santos, University of Minho

**On quasi-linear variational and quasi-variational inequalities with fractional gradient constraint**

We extend the existence of generalized Lagrange multipliers for the  $\sigma$ -gradient constrained problem for a class of quasi-linear monotone operators obtained previously for stationary second order linear operators involving the distributional Riesz fractional gradient. In the case of coercive quasi-linear operators, using the continuous dependence of the variational inequality solution, we also extend the existence of solutions to quasi-variational inequalities when the threshold of the fractional gradient constraint is given by a nonlocal operator depending also on the solution. (Joint work with A. Azevedo and J.F. Rodrigues)

Eduardo Teixeira, University of Central Florida

**On a class of free boundary problems involving diffusion weights**

I will discuss a class of free boundary problems ruled by singular/degenerate diffusion operators and their connections to nonlocal free boundary problems. Towards the end, I will report how such geometric insights can be used to tackle issues in the classical fully nonlinear regularity theory.

Catharine W.K. Lo, University of Lisbon

**On a Class of Nonlocal Obstacle Type Problems**

We consider one or two obstacles problems, and the  $N$ -membranes problem for a non-symmetric Dirichlet form in convex sets of  $H_0^s(\Omega)$ ,  $0 < s < 1$ . We extend several results, such as the weak maximum principle, comparison properties, approximation by bounded penalization, and also the Lewy-Stampacchia inequalities. This provides regularity of the solutions, in  $L^\infty(\Omega)$  in the general case and local regularity in  $W_{loc}^{2s,p}(\Omega)$  in the special case corresponding to the fractional  $s$ -Laplacian obstacle problem, which can be written with the distributional Riesz fractional  $s$ -derivatives. The novel results are complemented with the extension of the Lewy-Stampacchia inequalities to the order dual of  $H_0^s(\Omega)$  and some remarks on the associated  $s$ -capacity and the  $s$ -nonlocal obstacle problem. This is a joint work with J.-F. Rodrigues.

Harbir Antil, George Mason University

**Approximation of Fractional Harmonic Maps**

This talk addresses the approximation of fractional harmonic maps. Besides a unit-length constraint, one has to tackle the difficulty of nonlocality. We establish weak compactness results for critical points of the fractional Dirichlet energy on unit-length vector fields. We devise and analyze numerical methods for the approximation of various partial differential equations related to fractional harmonic maps. The compactness results imply the convergence of numerical approximations. Numerical examples on spin chain dynamics and point defects are presented to demonstrate the effectiveness of the proposed methods. This is a joint work with Soeren Bartels (Freiburg, Germany) and Armin Schikorra (Pittsburgh, USA)



Thu 16:15–18:15

## Minisymposium

Thu 16:15–18:15 Room 2

**FBP in the life sciences**

Organizers: A. Friedman, L. Berlyand

Avner Friedman, The Ohio State University

**A free boundary problem for fungal infection**

Fungi are cells found in commensal residence, on skin and on mucosal surfaces of the human body, including the digestive tract, but some species are pathogenic. Fungal infection may spread into deep-seated organs causing life-threatening situations. Effective defense against fungal infection requires coordinated response by the innate and adaptive immune systems. We represent the densities of the adaptive immune response by neutrophils,  $N$ , the adaptive immune response by T cells,  $T$ , and the fungal density by  $F$ , and introduce the following parameters:  $A$  = density of N cells and  $B$  = density of T cells in steady health state;  $a$ ,  $b$  the strengths of the innate and adaptive immune systems, respectively, in response to fungal infection; and  $C$  = growth rate of fungi. Then  $J = aA + bB - C$  represents the total balance of the immune response against fungal infection. We developed a mathematical model of fungal infection consisting of a PDE system for  $(F, N, T)$  in the fungal domains  $D(t)$ , where the free boundary moves in the normal direction  $n$  with velocity  $V(t) = h(-dF/dn)$ , where  $h(s)$  is a linearly-bounded monotone increasing function. In the radially-symmetric case, with  $D(t) = r < R(t)$  and free boundary  $r = R(t)$ , we prove global existence and uniqueness, with  $dR(t) > 0$ , and the following results: If  $J < 0$  then  $R(t)$  remains bounded and  $F$  converge to 0 as  $t$  increases to infinity, If  $J > 0$  and  $R(t)$  increases to infinity as  $t$  increases to infinity then  $(F, N, T)$  converges to  $(F^*, N^*, T^*)$  as  $t$  increases to infinity. If  $J > 0$  and  $R(0) > R^*$ , then  $R(t)$  converges to infinity as  $t$  increase to infinity. The quantities  $F^*, N^*, T^*$  and  $R^*$  are given explicitly by the parameters of the PDE system. This is a joint work with King-Yeung Lam.

Bei Hu, University of Notre Dame

**A Free Boundary Problem for modeling Plaques in the Artery – Recent progress**

Atherosclerosis is a leading cause of death worldwide; it originates from a plaque which builds up in the artery. We considered a simplified model of plaque growth involving LDL and HDL cholesterol, macrophages and foam cells, which satisfy a coupled system of PDEs with a free boundary, the interface between the plaque and the blood flow. In an earlier work (with Avner Friedman and Wenrui Hao), we proved that there exist small radially symmetric stationary plaques and established a sharp condition that ensures their stability. In our recent work (with Xinyue Zhao), we look for the existence of non-radially symmetric stationary solutions. The absence of an explicit radially symmetric stationary solution presents a big challenge to verify the Crandall-Rabinowitz theorem; through asymptotic expansion, we extend the analysis to establish a finite branch of symmetry-breaking stationary solutions which bifurcate from the radially symmetric solutions. Since plaque is unlikely to be strictly radially symmetric, our result would be useful to explain the asymmetric shapes of plaque.

Adrian Lam, The Ohio State University

**A free boundary problem of rheumatoid arthritis**

Rheumatoid arthritis is an autoimmune disease characterized by inflammation in the synovial fluid within the synovial joint connecting two contiguous bony surfaces. The inflammation diffuses into the cartilage adjacent to each of the bony surfaces, resulting in their gradual destruction. The interface between the cartilage and the synovial fluid is an evolving free boundary.

We consider a two-phase free boundary problem based on a simplified model of rheumatoid arthritis. In the model's simplified geometry, the free boundary is a function  $x=R(t)$  and the cartilage thickness is  $L-R(t)$ . We prove global existence and uniqueness of a solution, and derive qualitative properties of the free boundary. It is proved that the free boundary increases in time, and the cartilage shrinks to zero as time approaches infinity, even under treatment by a drug. It is also shown in the reduced one-phased problem, with cartilage alone, that a larger prescribed inflammation function leads to a faster destruction of the cartilage. This is joint work with Avner Friedman.

Tim Laux, University of Bonn

**The Hele-Shaw flow as the sharp-interface limit of the Cahn–Hilliard equation with disparate mobilities**

The Cahn–Hilliard equation is a well-studied phase-field model. In this talk, I will present a rigorous derivation of its sharp-interface limit with a mobility function that degenerates in one of the two phases. The proof is based on the interpretation of the PDE as a Wasserstein gradient flow. The limiting PDE is the well-known Hele-Shaw flow, which governs the slow movement of the free boundary of a drop of water in the narrow gap between two parallel horizontal plates. Our proof is flexible enough to accommodate other effects such as external forces. This is joint work with Milan Krömer (University of Bonn).

## Contributed Minisymposium

Thu 16:15–18:15 Room 1

**Geometric evolution of interfaces and transition layers (Part I)**

Organizers: G. Wheeler, P. Rybka

Marco Morandotti, Politecnico di Torino

**Analysis of a perturbed Cahn–Hilliard model for Langmuir–Blodgett films**

A one-dimensional evolution equation including a transport term is considered; it models a process of thin films deposition. Existence and uniqueness of solutions, together with continuous dependence on the initial data and an energy equality are proved by combining a minimizing movement scheme with a fixed-point argument. Finally, it is shown that, when the contribution of the transport term is small, the equation possesses a global attractor and converges to a purely diffusive Cahn–Hilliard equation. This is joint work with Marco Bonacini and Elisa Davoli.

Danielle Hilhorst, CNRS and University Paris-Saclay

**Singular limit for a stochastic Allen–Cahn equation with nonlinear diffusion**

We study a stochastic Allen–Cahn equation with nonlinear diffusion and a mild random noise on a bounded domain in  $\mathbb{R}^n$ . We consider initial data that satisfy some non-degeneracy conditions, and prove that steep transition layers develop within a very short time. We then study the motion of these transition layers and derive a stochastic motion law for the sharp interface limit. This is joint work with Perla El Kettani, Yong Joung Kim and Hyunjoon Park.

Katarzyna Ryszewska, Warsaw University of Technology

**A space-fractional Stefan problem**

In this talk we will consider a non-local in space, one-phase one-dimensional Stefan problem, where the diffusive flux takes the form of the fractional Caputo derivative. The motivation for studying such a problem originates from modelling the diffusion and mass transport in heterogeneous media. A typical example of such phenomenon is a sub-surface water motion. During the talk we will present the result concerning existence of regular solution to this problem. The proof relies mainly on analytic evolution operator theory and fixed point argument.

Mikołaj Sierżęga, University of Warsaw

**Optimal Liouville theorem for a semilinear Ornstein–Uhlenbeck equation**

In their seminal 1985 paper Giga and Kohn analysed the blow-up behaviour of the subcritical Fujita equation through a Liouville theorem for an associated elliptic equation of Ornstein–Uhlenbeck type. In a subsequent work Giga provided a conditional extension of this Liouville theorem to a natural broader class of semilinear Ornstein–Uhlenbeck equations and posed a question of its unconditional validity, i.e. in the class of bounded entire solutions. In this talk we will show that indeed such a result holds. The key ingredient of the demonstration relies on a generalisation of the Rellich–Pohozaev type argument which employs a family of special multipliers based on Kummer functions. Even though the problem is posed on the whole space this generalisation takes into account properties of the domain and provides a blueprint for analogous problems involving boundaries.

## Contributed Session

Thu 16:15–18:15 Room 3

**Contributed session (Part IV)**

Rolf Krause, USI - Università della Svizzera italiana

**A recursive multilevel trust region method for phase-field models**

We present and discuss non-linear multilevel methods for the numerical solution of phase-field models for fracture. Phase field models are computationally challenging due to large number of unknowns required for resolving the crack zone and the ill conditioning caused by local changes in the damage variable. We employ a recursive multilevel trust region method, which combines global convergence with multigrid efficiency. A crucial aspect is the design of level dependent objective functions, as the underlying model relies on the mesh dependent parameters. We introduce solution dependent objective functions that combine fine level description of the crack path with the coarse level discretization. Numerical examples in are presented.

Irina Denisova, Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences

**A problem governing the rotation of a two-phase capillary liquid drop**

We consider stability problem of a uniformly rotating liquid mass consisting of two viscous incompressible capillary self-gravitating liquids separated by a closed interface and bounded outside by a free surface. The flow is governed by a free boundary problem for the Navier–Stokes system with respect to unknown perturbations of velocity vector field and pressure function. An additional condition is the fact that boundary speed is equal to the normal velocity on the free boundary and on the interface. We obtain unique solvability of a linear problem, outline ways to prove the global solvability of the nonlinear problem provided that the initial data and the rotation speed are small, as well as the proximity of unknown surfaces to certain axisymmetric equilibrium figures.

Sebastian Aland, TU Freiberg

**A combined sharp/diffuse interface approach for wetting of elastic substrates**

The interaction of an elastic substrate with liquid droplets is at small length scales dominated by surface tension forces. We present a combination of a sharp and diffuse interface model to simulate this scenario. The fluid-fluid interface is represented by a diffuse layer to regularize the contact line singularity. The solid-fluid interface is represented by a matched finite-element grid and moved in the typical Arbitrary-Lagrangian-Eulerian way. As all equations are formulated in a Eulerian frame of reference, we obtain a single momentum equation including the solid and both fluid materials, which is very simple to solve monolithically. We illustrate the numerical robustness of this novel method and compare it to the recent experimental and numerical results.

Lucas Daniel Wittwer, HTW Dresden

**Simulations of an active surface immersed in viscous fluids**

The surface of biological cells is connected to the cell cortex - a thin layer of active material. This layer exerts an active contractile tension which regulates cellular shape by deforming the surface. The strength of the active tension is controlled by the concentration of force-generating molecules. Advective transport of these molecules leads to a complex interplay of surface deformation, hydrodynamics and molecule concentration which gives rise to pattern formation and self-organized shape dynamics. In this talk, we present a numerical model to

simulate such an active viscous surface immersed in viscous fluids. We show the resulting patterning and cell shape dynamics as well as the flow profiles in the two bulk phases and discuss implications to the functioning of biological cells.

Contributed Session

Thu 16:15–18:15 Room 4

**Contributed session (Part V)**

Guozhi Dong, Humboldt-Universität zu Berlin

**A class of second order geometric quasilinear hyperbolic PDEs and their application**

We propose a class of novel second-order nonlinear hyperbolic PDEs inspired by the recent development of second-order dynamics for accelerating the energy decay of gradient flows. We put particular focus on a second-order total variation flow and a second-order level set mean curvature flow, both with damping terms weighted by some chosen parameters. We will show some analytical results on the proposed equations, i.e., existence of solutions, uniqueness, and also some numerical aspects from the applications in imaging point of view, e.g., denoising, correcting displacement errors. This is a joint work with Michael Hintermueller and Ye Zhang.

Vladimir Vasilyev, Belgorod State National Research University

**On Some Elliptic Boundary Value Problems in Canonical Domains**

We study solutions of a boundary value problem for elliptic pseudo-differential equation  $(Au)(x) = 0$ ,  $x \in C_+^a$ , where  $C_+^a$  is a certain cone in  $m$ -dimensional space  $\mathbb{R}^m$ , with a certain non-local additional condition. Using results and methods from “Wave factorization of elliptic symbols, Kluwer Acad. Publ., 2000” we study behavior of a solution for this problem under  $a \rightarrow \infty$ , it corresponds to such case when two-dimensional cone transforms to a ray. Let us note that if  $a \rightarrow 0$ , it corresponds to the half-space  $\mathbb{R}_+^m$ . The main conclusion is the following: the boundary value problem has a limit under  $a \rightarrow \infty$  only if the boundary function  $g(x_1)$  satisfies certain functional one-dimensional singular integral equation.

Seongmin Jeon, Purdue University

**Almost minimizers for the thin obstacle problem with variable coefficients**

In this talk, we will consider almost minimizers for the thin obstacle problem with variable Hölder continuous coefficients and zero thin obstacle, and discuss their regularity properties. Under an additional assumption of quasisymmetry, we will establish the regularity of the regular set and a structural theorem on the singular set. The proofs are based on the generalization of Weiss- and Almgren-type monotonicity formulas for almost minimizers established earlier in the case of constant coefficients. This is joint work with Arshak Petrosyan and Mariana Smit Vega Garcia.

Fri 09:50–11:50

## Contributed Minisymposium

Fri 09:50–11:50 Room 1

**Geometric evolution of interfaces and transition layers (Part II)**

Organizers: G. Wheeler, P. Rybka

Glen Wheeler, University of Wollongong

**On Chen's flow**

In this talk we quickly survey what is currently known for Chen's flow, and discuss some very recent results. Chen's flow is the biharmonic heat flow for immersions, where the velocity is given by the rough Laplacian of the mean curvature vector. This operator is known as Chen's biharmonic operator and the solutions to the elliptic problem are called biharmonic submanifolds. The flow itself is very similar to the mean curvature flow (this is essentially the content of Chen's conjecture), however proving this requires quite different strategies compared to the mean curvature flow. We focus on new results available in low dimensions - curves, surfaces, and 4-manifolds.

James McCoy, University of Newcastle

**High order curvature flow of plane curves with generalised Neumann boundary conditions**

We consider the parabolic polyharmonic diffusion and  $L^2$ -gradient flows of the  $m$ -th arclength derivative of curvature for regular curves evolving with generalised Neumann boundary conditions. In the polyharmonic case, we prove that if the curvature of the initial curve is small in  $L^2$ , then the evolving curve converges exponentially in the  $C^\infty$  topology to a straight horizontal line segment. The same behaviour is shown for the  $L^2$ -gradient flow provided the energy of the initial curve is sufficiently small. In each case the smallness conditions depend only on  $m$ . This is joint work with Glen Wheeler and Yuhan Wu.

Simon Blatt, Paris Lodron University Salzburg

**On the Negative Gradient Flow of the  $p$ -Energy**

This talk will be about ongoing work on the negative  $L^2$  gradient flow of  $p$ -elastic energies for curves. After an overview over known results we will discuss short- and longtime existence for these evolution equations using de Giorgi's method of minimizing movements. Apart from that the essential new tool in our approach is to write the evolving curves as approximate normal graphs over a fixed smooth curves. In this way we can prove a lower bound for the time of existence that only depends on the  $W^{2,p}$  norm of the initial curve. We will shortly discuss the pros and cons of this method and present some open problems.

Piotr Rybka, University of Warsaw

**A sixth order Cahn–Hilliard type equation with a small deposition rate**

We consider a one-dimensional sixth order Cahn–Hilliard type equation describing the evolution of a growing crystalline surface that undergoes faceting,

$$u_t = \delta u u_x + \Delta^3 u - (u^3 - u)_{xxxx}$$

with periodic boundary conditions. This equation contains a destabilizing quadratic term corresponding to a deposition rate. The well-posedness of this equation is known for periodic boundary conditions. Existence of a global compact attractor is also known. We study the case when the deposition rate is small. We show that in this case the equation has a gradient flow structure in an appropriate sense. Using this observation we prove stabilization of solutions.

## Contributed Session

Fri 09:50–11:50 Room 2

**Contributed session (Part VI)**

Enrico Valdinoci, University of Western Australia

**The stickiness phenomenon for nonlocal minimal surfaces**

We will present some recent results related to the boundary behavior of nonlocal minimal surfaces, especially connected with boundary discontinuities, stickiness to the reference domain, yin-yang results, multiplicity of minimizers, and topological changes.

Subas Acharya, University of Texas at Dallas

**Free Boundary Problem with Real Options**

We consider a Free Boundary Problem (FBP) associated with a real options problem. The real options problem is posed as a stochastic optimal control problem. The investment strategy (control), involves a one-time option to expand and a one-time option to terminate the project. The timing and amount of the investment are parameters to be optimized to maximize the profit. This stochastic optimization problem is reduced to a variational inequality (VI) and then to a FBP for a PDE. We use a specific substitution for the value function to reduce the PDE into a FBP for ODE. The focus of this work is on deriving sufficient conditions for solvability of the VI and providing a characterization of the solution (the value function), for a general form of the nonlinear investment cost.

Xin Liu, Weierstrass Institute

**Second law of thermodynamics and bounded entropy solutions in the compressible Navier–Stokes system**

Our goal is to investigate the existence of global-in-time solutions with bounded entropy to the compressible Navier–Stokes system. Previous study of Cauchy problems motivates the free boundary problem, where the flows connect to vacuum with continuous density on the moving boundary. We mainly focus on the spherically symmetric solutions. Two results will be presented. 1. Global solutions to isentropic flows with or without density jump on the moving boundary; 2. A class of self-similar solutions are presented, and the bounded entropy solutions are obtained with initial data perturbed around such self-similar data.

Yvonne Stokes, University of Adelaide

**Asymptotic modelling of drawing of micro-structured optical fibres**

An efficient asymptotic model for the drawing of fibres with arbitrary cross-sectional geometry will be described. ODEs describe the change in area of the cross-section and the temperature from preform to fibre. In general, these are coupled with a 2D transverse-flow problem describing the evolution of the geometry in the cross-section due to surface tension and pressure. The accuracy of the model will be demonstrated by comparison with experiments and finite-element simulations.

Fri 12:45–14:45

Minisymposium

Fri 12:45–14:45 Room 1

**Free boundary problems in cell biology (Part II)**

Organizers: C. Gräser, M. Röger

**Chandrasekhar Venkataraman, University of Sussex  
Multiscale modelling, analysis and simulation of cell signalling processes**

We consider homogenisation of a model for cell signalling processes in biological tissues. Such signalling processes are the primary mechanism by which cells interact and respond to external stimuli. Hence they play an important role in the majority of cell biological phenomena. The signalling model we consider includes diffusion and nonlinear reactions on the cell surfaces, and both inter- and intracellular signalling. We derive, under the assumption of a periodic cell distribution, the equations satisfied in the limit as the cell number tends to infinity with the volume fraction of tissue occupied by the cells constant. Furthermore, we present a finite element method for the limiting two-scale bulk surface system and report on numerical simulations of the model equations.

**Philip Herbert, University of Koblenz-Landau  
Surface PDE models for membrane mediated particle interactions on near-spherical or near-tubular biomembranes**

We consider a hybrid model of a biomembrane with attached proteins. We investigate the arrangement of proteins with respect to membrane mediated interactions. The membrane is represented by a near-spherical or near-tubular surface and proteins by rigid bodies which attach to the membrane at finitely many points. The model is based on tractable quadratic energies which are perturbations of the Canham-Helfrich energy with point constraints. Well-posedness of the minimisation problem for the shape of the membrane and finite element approximations will be described. We prove differentiability of this energy with respect to the location of attached particles and give a formula for the derivative. Results will be illustrated by numerical experiments. Joint work with C.M. Elliott and C. Gräser

**Diane Peurichard, Inria Paris  
Multi-scale approach for a model of tumor growth: from short-range repulsion to Hele-Shaw problems**

In this talk, we investigate the multi-scale link for a model of tumor growth. We start from a microscopic model where cells are modelled as 2D spheres undergoing short range repulsion and cell division. We derive the associated macroscopic dynamics leading to a porous media type equation. As the macroscopic equation obtained through usual derivation method fails at providing the correct qualitative behavior, we propose a modified version of the macroscopic equation introducing a density threshold for the repulsion. We numerically validate the new formulation by comparing the solutions of the micro- and macro- dynamics. Moreover, we study the asymptotic behavior of the dynamics as the repulsion between cells becomes singular (leading to non-overlapping constraints in the microscopic model)

Minisymposium

Fri 12:45–14:45 Room 2

**Surface PDEs (Part II)**

Organizers: C. Venkataraman, T. Ranner

**Axel Voigt, TU Dresden  
Fluid deformable surfaces - a numerical approach by surface FEM**

Lipid membranes are examples of fluid deformable surfaces. They can be viewed as two-dimensional viscous fluids with bending elasticity. With this solid-fluid duality any shape change contributes to tangential flow and vice versa any tangential flow on a curved surface induces shape deformations. This tight coupling between shape and flow makes curvature a natural element of the governing equations. We derive the governing equations as a thin-film limit of a generalized Navier–Stokes equation, propose a numerical approach by finite elements and show numerical results, which can serve as benchmark problems.

**Paola Pozzi, University of Duisburg-Essen  
On a converging FE-scheme for motion by curvature of a network with a triple junction.**

In this talk I will present a new semi-discrete finite element scheme for the evolution of three parametrised curves that are connected by a triple junction and move by curvature flow. At the triple junction conditions are imposed on the angles at which the curves meet. I will provide ideas underlying the convergence analysis and show some numerical tests. This is joint work with B. Stinner.

**Balázs Kovács, University of Regensburg  
A convergent evolving finite element algorithm for Willmore flow of closed surfaces**

We will sketch a proof of convergence for a finite element semi-discretisation of Willmore flow of closed two-dimensional surfaces. As our algorithm for mean curvature flow, the proposed method discretizes evolution equations for the normal vector and mean curvature, together with the velocity law. This numerical method admits a convergence analysis, which combines stability estimates and consistency estimates to yield optimal-order  $H^1$ -norm semi-discrete error bounds. The stability analysis is based on the matrix-vector formulation of the finite element method and does not use geometric arguments. Stability is proven using a non-standard version of energy estimates, which we will sketch during the talk. We will also present various numerical experiments.

**Matthias Röger, TU Dortmund  
A bulk-surface Gierer-Meinhardt model**

We consider a coupled system of a diffusion equation in a volume coupled to a Gierer-Meinhardt type system on the boundary. We prove existence of solutions and present numerical simulations.

## Minisymposium

Fri 12:45–14:45 Room 3

**Asymptotic approaches to interface dynamics (Part II)**

Organizers: J. José López Velázquez, J. King

Jonas Jansen, Universität Bonn

**Homogenisation of a nonlinear Poisson equation in perforated domains and a free-boundary problem**

One important problem in homogenisation is to describe an effective theory for electric charges in a material with many tiny holes or the flow of a fluid around (potentially moving) particles. To obtain such a theory, one considers randomly distributed holes and sends the number of holes to infinity. Depending on the scaling behaviour of the radii of the holes, one obtains different deterministic limiting equations, including - in the linear case - the famous Cioranescu-Murat limit. In this talk, I will discuss homogenisation phenomena of the nonlinear equation  $-\Delta u_m + \lambda_m |u_m|^{p-1} u_m = 0$  in a perforated domain in the three-dimensional torus with deterministic Dirichlet boundary conditions on the holes. I will discuss different scaling limits and show how a free-boundary problem arises as homogenisation limit.

Mark Blyth, University of East Anglia

**Asymptotic solutions for free-surface flow over localised topography**

Steady free-surface flow over a localised bottom topography is examined at critical Froude number ( $F = 1$ ). In this case the forced KdV equation that applies for small amplitude topography reduces to the inhomogeneous nonlinear ordinary differential equation for the free-surface disturbance  $u(x)$ ,

$$u'' + u^2 = \alpha f(x),$$

where  $\alpha$  is a parameter. The rate of decay of the forcing function  $f(x)$  in the far field is important and it has a strong influence on the structure of the solution space. Numerical calculations for a Gaussian forcing, with exponential far-field decay, suggest that there exists an infinite number of distinct solution branches each of which terminates at some value of  $\alpha$ . We provide an asymptotic description of the solution valid for large  $\alpha$  on any given branch following a standard boundary-layer type of approach augmented by a method for glueing homoclinic connections. We also present a general asymptotic description of the solutions near to a termination point.. Solutions for topographies whose solution branches do not terminate, including a Lorentzian forcing, will also be briefly discussed.

Linda Cummings, New Jersey Institute of Technology

**Asymptotic thermal modeling of droplet assembly in nanoscale molten metal films**

We consider a thin metal film on a thermally conductive substrate exposed to an external heat source in a setup where the heat absorption depends on the local film thickness. Our focus is on modeling film evolution while the film is molten. The film geometry modifies local heat flow, which in turn may influence the film surface evolution through thermal variation of material properties. We use asymptotic analysis to develop a thermal model that is accurate, computationally efficient, and that accounts for the heat flow in both the in-plane and out-of-plane directions. We apply this model to describe metal films of nanoscale thickness exposed to heating and melting by laser pulses, a setup commonly used for self and directed assembly of various metal geometries via dewetting while the films are in the liquid phase. We find that thermal effects play an important role, and in particular that the inclusion

of temperature dependence in the metal viscosity modifies the time scale of the evolution significantly. The thickness, thermal conductivity, and rate of heat loss of the underlying substrate are shown to be crucial in accurately modeling film temperatures and subsequent phase changes in the film. Since in many cases the substrate cools the film, modifications to the substrate temperature may induce different dewetting speeds via temperature dependent viscosity of the film. We show via 3D GPU simulations that this may result in various frozen film patterns since full dewetting may not occur while the film is in the liquid phase. This research was supported by NSF CBET-1604351, NSF-DMS-1815613 and by CNMS2020-A-00110.

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